COMPOSITE MATERIALS

SYLLABUS

UNIT-I

Introduction: Definitions, Composites, Reinforcements and matrices, Types of reinforcements, Types of matrices, Types of composites, Carbon Fibre composites, Properties of composites in comparison with standard materials, Applications of metal, ceramic and polymer matrix composites.

UNIT-II

Manufacturing methods : Hand and spray lay - up, injection molding, resin injection, filament winding, pultrusion, centrifugal casting and prepregs. Fibre/Matrix Interface, mechanical. Measurement of interface strength. Characterization of systems; carbon fibre/epoxy, glass fibre/polyester, etc.

UNIT-III

Mechanical Properties -Stiffness and Strength: Geometrical aspects – volume and weight fraction. Unidirectional continuous fibre, discontinuous fibers, Short fiber systems, woven reinforcements –Mechanical Testing: Determination of stiffness and strengths of unidirectional composites; tension, compression, flexure and shear.

UNIT-IV

Laminates : Plate Stiffness and Compliance, Assumptions, Strains, Stress Resultants, Plate Stiffness and Compliance, Computation of Stresses, Types of Laminates -, Symmetric Laminates, Anti-symmetric Laminate, Balanced Laminate, Quasi-isotropic Laminates, Cross-ply Laminate, Angle-ply Laminate. Orthotropic Laminate, Laminate Moduli, Hygrothermal Stresses.

UNIT-V

Joining Methods and Failure Theories : Joining –Advantages and disadvantages of adhesive and mechanically fastened joints. Typical bond strengths and test procedures.

UNIT-I

INTRODUCTION

Introduction to Composite Materials

A composite material is a material that consists of one or more discontinuous components (particles/fibres/reinforcement) that are placed in a continuous medium (matrix).In a fibre composite the matrix binds together the fibres, transfers loads between the fibres and protects them from the environment and external damage.

Composite materials, or shortened to composites, are microscopic or macroscopic combinations of two or more distinct engineered materials (those with different physical and/or chemical properties) with a recognizable interface between them in the finished product. For structural applications, the definition can be restricted to include those materials that consist of a reinforcing phase such as fibers or particles supported by a binder or matrix phase. Wood composites are commonly seen examples of composite materials.

Other features of composites include the following: (1) The distribution of materials in the composite is controlled by mechanical means. (2) The term composite is usually reserved for materials in which distinct phases are separated on a scale larger than atomic,and in which the composite's mechanical properties are significantly altered from those of the constituent components. (3) The composite can be regarded as a combination of two or more materials that are used in combination to rectify a weakness in one material by a strength in another. (4) A recently developed concept of composites is that the composite should not only be a combination of two materials, but the combination should have its own distinctive properties. In terms of strength, heat resistance, or some other desired characteristic, the composite must be better than either component alone.

Composites were developed because no single, homogeneous structural material could be found that had all of the desired characteristics for a given application. Fiber-reinforced composites were first developed to replace aluminum alloys, which provide high strength and fairly high stiffness at low weight but are subject to corrosion and fatigue.

An example of a composite material is a glass-reinforced plastic fishing rod in which glass fibers are placed in an epoxy matrix. Fine individual glass fibers are characterized by their high tensile stiffnesses and very high tensile strengths, but because of theirsmall diameters, have very small bending stiffnesses. If the rod were made only of epoxy plastic, it would have good bending stiffness, but poor tensile properties. When the fibers are placed in the epoxy plastic, however, the resultant structure has high tensile stiffness, high tensile strength, and high bending stiffness.

The discontinuous filler phase in a composite is usually stiffer or stronger than the binder phase. There must be a substantial volume fraction of the reinforcing phase (about10%) present to provide reinforcement. Examples do exist, however, of composites where the discontinuous phase is more compliant and ductile than the matrix.

Natural composites include wood and bone. Wood is a composite of cellulose and lignin. Cellulose fibers are strong in tension and are flexible. Lignin cements these fibers together to make them stiff. Bone is a composite of strong but soft collagen (a protein) and hard but brittle apatite (a mineral).

Particulate composites
  Various geometrical shapes (cubes, spheres, flakes, etc.)
  Various materials (rubber, metal, plastics, etc.)
  Have generally low strength.
  Will not be treated further in this course.

Particle reinforced metal matrix composites are now being produced commerically, and in this paper the current status of these materials is reviewed. The different types of reinforcement being used, together with the alternative processing methods, are discussed. Depending on the initial processing method, different factors have to be taken into consideration to produce a high quality billet. With powder metallurgy processing, the composition of the matrix and the type of reinforcement are independent of one another. However, in molten metal processing they are intimately linked in terms of the different reactivities which occur between reinforcement and matrix in the molten state. The factors controlling the distribution of reinforcement are also dependent on the initial processing method. Secondary fabrication methods, such as extrusion and rolling, are essential in processing composites produced by powder metallurgy, since they are required to consolidate the composite fully. Other methods, such as spray casting, molten metal infiltration, and molten metal mixing give an essentially fully consolidated product directly, but extrusion, etc., can improve the properties by modifying the reinforcement distribution. The mechanical properties obtained in metal matrix composites are dependent on a wide range of factors, and the present understanding, and areas requiring further study, are discussed. The successful commercial production of metal matrix composites will finally depend on their cost effectiveness for different applications. This requires optimum methods of processing, machining, and recycling, and the routes being developed to achieve this are considered.

Fibre composites
Discontinuous or Continuous

Classification of Composites:

- Matrices:

Organic Matrix Composites (OMCs)
Polymer Matrix Composites (PMCs)
carbon-carbon composites
Metal Matrix Composites (MMCs)
Ceramic Matrix Composites (CMCs)

- Reinforcements:

Fibres reinforced composites
Laminar composites
Particulate composites

Carbon Fiber is a polymer and is sometimes known as graphite fiber. It is a very strong material that is also very lightweight. Carbon fiber is five-times stronger than steel and twice as stiff. Though carbon fiber is stronger and stiffer than steel, it is lighter than steel; making it the ideal manufacturing material for many parts. These are just a few reasons why carbon fiber is favored by engineers and designers for manufacturing.

Properties of Composite Materials:

1. **High Strength to Weight Ratio:** Fibre composites are extremely strong for their weight. By refining the laminate many characteristics can be enhanced. A common laminate of say 3mm Chopped strand mat, is quite flexible compared to say a 3 mm ply. However it will bend a long way more than the ply before yielding. Stiffness should not be confused with Strength. A carbon fibre laminate on the other hand, will have a stiffness of many times that of mild steel of the same thickness, increased ultimate strength, yet only be less than 1/4 of it's weight.
2. **Lightweight:** A standard Fibreglass laminate has a specific gravity in the region of 1.5, compared to Alloy of 2.7 or steel of 7.8. When you then start looking at Carbon laminates, strengths can be many times that of steel, but only a fraction of the weight. A DVD case lid was produced using carbon fibre to reduce the case's overall weight so that it could be carried as cabin baggage whilst traveling, and for improved security. It was used by support crew for the All Blacks during their 1999 Rugby World Cup campaign.
3. **Fire Resistance:** The ability for composites to withstand fire has been steadily improving over the years. There is two types of systems to be considered:
   Fire Retardant - Are self extinguishing laminates, usually made with chlorinated resins and additives such as Antimony trioxide. These release CO2 when burning so when the flame source is removed, the self extinguish.
   Fire Resistant - More difficult and made with the likes of Phenolic Resins. These are difficult to use, are cured with formaldehyde, and require a hi degree of post curing to achieve true fire resistance.
   Other materials are also becoming more readily available to be used as in tumescent layers, which expand and blanket the surface, preventing spread of flame. There is a paint on coating usually applied to the back of the product laminate, plus a thin fibre film to

go under the Gelcoat giving the outer surface a blanketing coat as well.

Fibreglass Developments Ltd produces a Fire Door as part of our **SteridorTM** range. Use of special Phenolic resin has allowed us to create the *only* fully tested Composite door in Australasia. Fire rated by BRANZ to 4 hours, this door is also approved by MAF as meeting all their Hygiene requirements.

4 **Electrical Properties:**Fibreglass Developments Ltd produced the Insulator Support straps for the Tranz Rail main trunk electrification. The straps, although only 4mm thick, meet the required loads of 22kN, as well as easily meeting insulation requirements.

5 **Chemical & Weathering Resistance:**Composite products have good weathering properties and resist the attack of a wide range of chemicals. This depends almost entirely on the resin used in manufacture, but by careful selection resistance to all but the most extreme conditions can be achieved. Because of this, composites are used in the manufacture of chemical storage tanks, pipes, chimneys and ducts, boat hulls and vehicle bodies. FDL manufactured architectural panels for the construction of the Auckland Marine Rescue Centre. Composite panels were chosen because of their ability to withstand salty sea side conditions without corrosion.

6 **Colour:**Almost any shade of any colour can be incorporated into the product during manufacture by pigmenting the gelcoat used. Costs are therefore reduced by no further finishing or painting. Soluble dyes can be used if a translucent product is desired. We do not however, recommend dark colours. These produce excessive heat on the surface which can lead to the surface deteriorating and showing print through, where the Resin matrix cures more and shrinks, bringing the fibres to the surface. In extreme cases delamination can occur.

7 **Translucency:**Polyester resins are widely used to manufacture translucent mouldings and sheets. Light transmission of up to 85% can be achieved.

8 **Design Flexibility:**Because of the versatility of composites, product design is only limited by your imagination.

9. **Low Thermal Conductivity:**Fibreglass Developments has been involved in the development and production of specialized meat containers which maintain prime cuts of chilled meat at the correct temperature for Export markets. They are manufactured using the RTM process, with special reinforcing and foam inserts.
10. **Manufacturing Economy:**Fibreglass Developments produces several models of fuel pump covers for Fuel quip. Fibreglass is an ideal material for producing items of this type for many reasons, including being very economical. Because of its versatile properties, fibreglass can be used in many varied applications.

Advantages:

- Lower density (20 to 40%)
- Higher directional mechanical properties (specific tensile strength (ratio of material strength to density) 4 times greater than that of steel and aluminium.
- Higher Fatigue endurance .
- Higher toughness than ceramics and glasses.
- Versatility and tailoring by design.
- Easy to machine.
- Can combine other properties (damping, corrosion).
- Cost.

Applications:

Mulch: In nature we see plants and trees drop leaves that accumulate at their bases. Every year, a new layer is added while the old layers start to decompose. This is leaf mold, and it is a form of compost. What nature is doing is providing a protective layer over the roots of plants. This layer of vegetative material protects the bare soil during the summer months by reducing soil temperature, suppressing weed growth and reducing soil moisture loss. Our compost can do the same thing in our gardens and landscapes.To prepare any area for mulching, first clear away grass or weeds that might grow up through the mulch. Make sure to remove the roots of tough perennial weeds such as ground ivy. When using compost as a mulch in flower beds, vegetable gardens, landscape beds, or lawns, screen the finished compost. A simple screen can be made using ½-inch mesh hardware cloth and attaching it to a wooden frame. Place the screen over a wheel barrow or other container and sift the compost into it. The large pieces left behind can go into your next compost pile as an activator, introducing the necessary microorganisms. Cover the garden or bed area with screened compost to a depth of one to two inches. If you apply compost on a lawn, be sure it is finely ground or sifted. You have less of a chance of smothering the lawn. You may want to use ¼-inch mesh hardware cloth. One way to incorporate the compost is to aerate the sod, then apply a 1/8-inch to ¼-inch covering of fine compost. Use a rake to distribute the compost into the crevices. When mulching around trees and shrubs, screening may not be necessary. This is really a matter of aesthetic desire on your part.

Soil Amendment: We have already talked about how compost helps soil, especially sandy and clay soils. When starting a new garden soil amending is recommended before you plant. It is so much easier to add compost now than it is after the garden is planted. Cover the garden area with 3 to 4 inches of compost and till it into the upper six inches of the soil. If your garden is already established and you want to incorporate compost deeply into the soil, your options are limited.

With perennials, every time you add a new plant to the garden or divide an existing one, add compost. With annuals, you can add compost every spring. Loosen up the entire area where annuals will be planted and work in compost. Around trees and shrubs add at planting time, mixing no more than 25 percent of soil volume. Some references say not to use any at all for fear that the roots will remain in the planting hole area and not grow out into the surrounding soil. Keeping the compost level at one-quarter of the total soil volume will not lead to this problem. If you’re concerned, use the compost as a mulch only.

Around existing trees it may be difficult to incorporate into the upper six inches of the soil. You can add compost by injecting nutrients the way professional arborists do. Drill 1-to 2-inch diameter holes 12 inches deep in the soil throughout the tree canopy and beyond at 18-inch spacing. Fill the bottom of each hole with recommended rates of dry fertilizer and then top off the holes with compost. For shrubs, the holes only need to be drilled 8 to 10 inches deep. This treatment should supply nutrients for two to three years.

Using Compost in Potting Mixes:

You can also blend fine-textured compost in potting mixtures. However, make sure the compost does not make up more than one quarter to one half of the potting mixture's volume. Plants growing in containers are entirely reliant on the water and nutrients provided in the potting mix. Compost is excellent for container growing mixes, because it stores moisture effectively and provides a variety of nutrients not typically supplied in commercial fertilizers or soil-free potting mixes. You still need to fertilize containers on a regular basis to provide the high volume of nutrients they need. Finely sifted compost can also be used in seed starting mixtures.

Compost Tea:

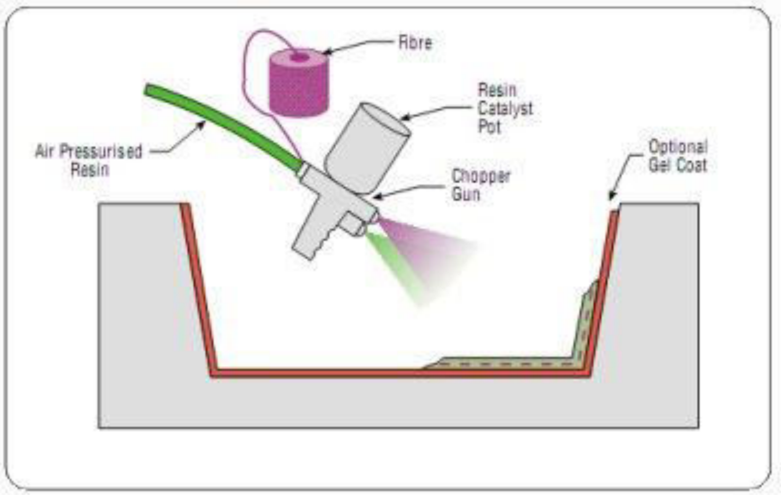
An old fashioned way of providing liquid fertilizer for plants is to brew compost tea. Similar to manure tea, compost tea gives your plants a good dose of nutrients. Compost tea works especially well for providing nutrients to new transplants and young seedlings. To make compost tea fill a burlap sack or an old pillow case with finished compost and secure the open end. Place in a tub, barrel, or watering can filled with water. Agitate for a few minutes and then let it steep for a few days. Water will leach out nutrients from the compost and the mixture will take on the color of tea. Spray or pour compost tea on and around plants. You can use the bag of compost for several batches. Afterwards, simply empty the bag's contents onto the garden.

UNIT-II

MANUFACTURING METHODS

Manufacturing of Composites

Spray Lay-Up



Description:

Fibre is chopped in a hand-held gun and fed into a spray of catalyzed resin directed at the mould. The deposited materials are left to cure under standard atmospheric conditions.

Material Options:

- Resins: Primarily polyester
- Fibres: Glass roving only
- Cores: None. These have to be incorporated separately

Typical Applications:

Simple enclosures, lightly loaded structural panels, e.g. caravan bodies, truck fairings, bathtubs, shower trays, some small dinghies.

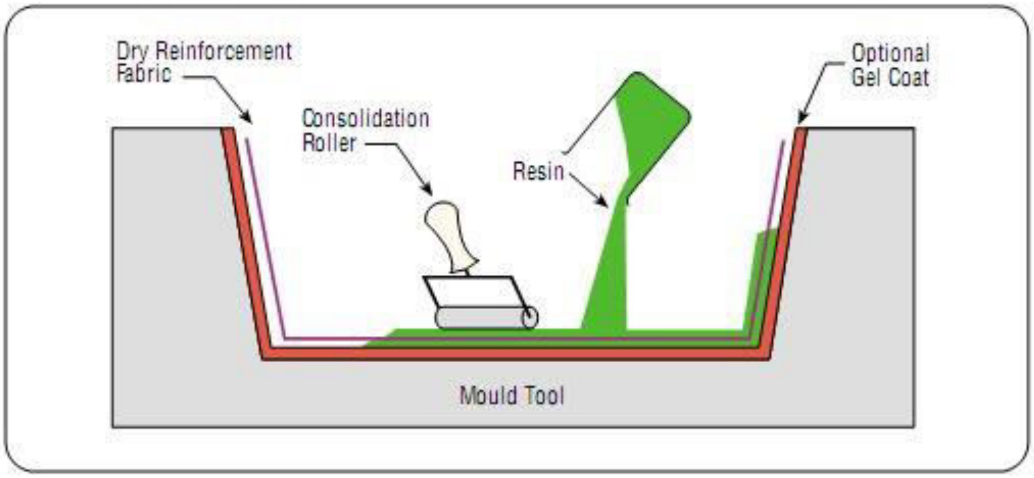
Advantages:

- Widely used for many years.
- Low cost way of quickly depositing fibre and resin.
- Low cost tooling.

Disadvantages:

- Laminates tend to be very resin-rich and therefore excessively heavy.
- Only short fibres are incorporated which severely limits the mechanical properties of the laminate.
- Resins need to be low in viscosity to be sprayable. This generally compromises their mechanical/thermal properties.
- The high styrene contents of spray lay-up resins generally mean that they have the potential to be more harmful and their lower viscosity means that they have an increased tendency to penetrate clothing.
- Limiting airborne styrene concentrations to legislated levels is becoming increasingly difficult.

Wet/Hand Lay-up



Description:

Resins are impregnated by hand into fibres which are in the form of woven, knitted, stitched or bonded fabrics. This is usually accomplished by rollers or brushes, with an increasing use of nip-roller type impregnators for forcing resin into the fabrics by means of rotating rollers and a bath of resin. Laminates are left to cure under standard atmospheric conditions.

Materials Options:

- Resins: Any, e.g. epoxy, polyester, vinylester, phenolic
- Fibres: Any, although heavy aramid fabrics can be hard to wet-out by hand.
- Cores: Any.

Typical Applications:

Standard wind-turbine blades, production boats, architectural mouldings.

Advantages:

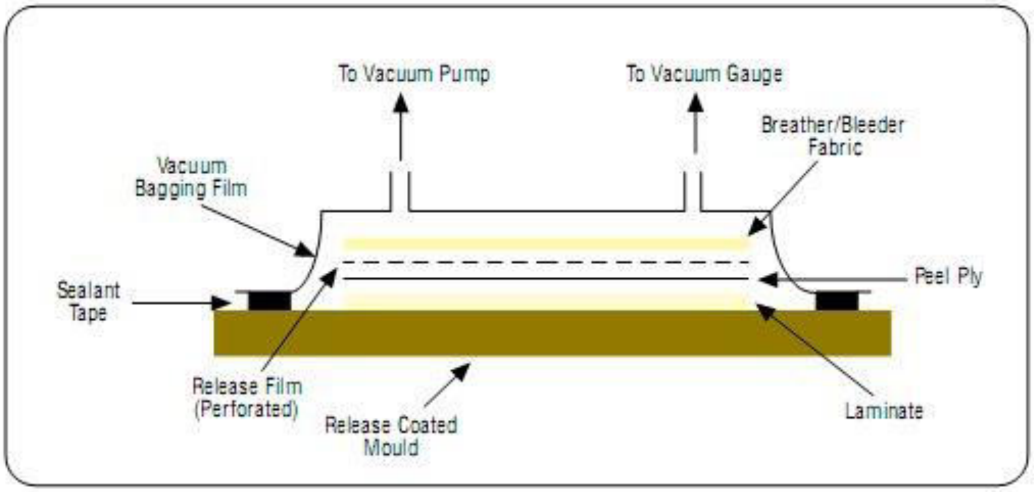
- Widely used for many years.
- Simple principles to teach.
- Low cost tooling, if room-temperature cure resins are used.
- Wide choice of suppliers and material types.
- Higher fibre contents and longer fibres than with spray lay-up.

Disadvantages:

- Resin mixing, laminate resin contents, and laminate quality are very dependent on the skills of laminators. Low resin content laminates cannot usually be achieved without the incorporation of excessive quantities of voids.

- Health and safety considerations of resins. The lower molecular weights of hand lay-up resins generally mean that they have the potential to be more harmful than higher molecular weight products. The lower viscosity of the resins also means that they have an increased tendency to penetrate clothing.
- Limiting airborne styrene concentrations to legislated levels from polyesters and vinylesters is becoming increasingly hard without expensive extraction systems.
- Resins need to be low in viscosity to be workable by hand. This generally compromises their mechanical/thermal properties due to the need for high diluent/styrene levels.

Vacuum Bagging



Description:

This is basically an extension of the wet lay-up process described above where pressure is applied to the laminate once laid-up in order to improve its consolidation. This is achieved by sealing a plastic film over the wet laid-up laminate and onto the tool. The air under the bag is extracted by a vacuum pump and thus up to one atmosphere of pressure can be applied to the laminate to consolidate it.

Materials Options:

- Resins: Primarily epoxy and phenolic. Polyesters and vinylesters may have problems due to excessive extraction of styrene from the resin by the vacuum pump.
- Fibres: The consolidation pressures mean that a variety of heavy fabrics can be wet-out.
- Cores: Any.

Typical Applications:

Large, one-off cruising boats, racecar components, core-bonding in production boats.

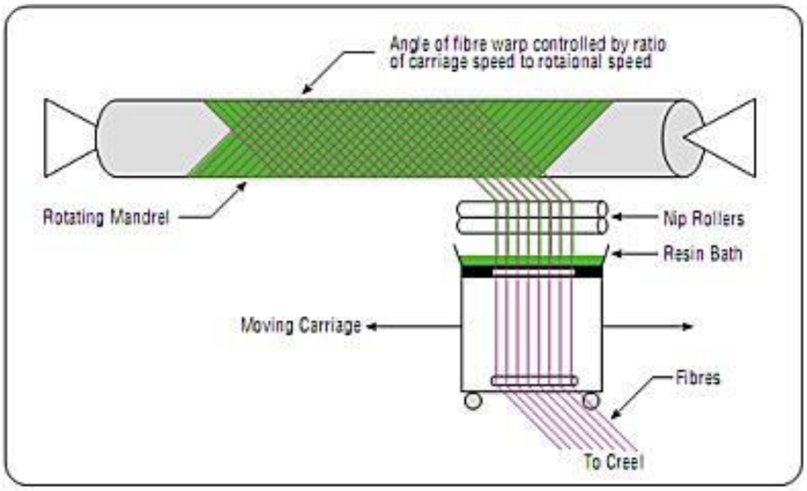
Advantages:

- Higher fibre content laminates can usually be achieved than with standard wet lay-up techniques.
- Lower void contents are achieved than with wet lay-up.
- Better fibre wet-out due to pressure and resin flow throughout structural fibres, with excess into bagging materials.
- Health and safety: The vacuum bag reduces the amount of volatiles emitted during cure.

Disadvantages:

- The extra process adds cost both in labour and in disposable bagging materials.
- A higher level of skill is required by the operators.
- Mixing and control of resin content still largely determined by operator skill.

Filament Winding:



Description:

This process is primarily used for hollow, generally circular or oval sectioned components, such as pipes and tanks. Fibre tows are passed through a resin bath before being wound onto a mandrel in a variety of orientations, controlled by the fibre feeding mechanism, and rate of rotation of the mandrel.

Materials Options:

- Resins: Any, e.g. epoxy, polyester, vinylester, phenolic
- Fibres: Any. The fibres are used straight from a creel and not woven or stitched into a fabric form
- Cores: Any, although components are usually single skin

Typical Applications:
Chemical storage tanks and pipelines, gas cylinders, fire-fighters breathing tanks

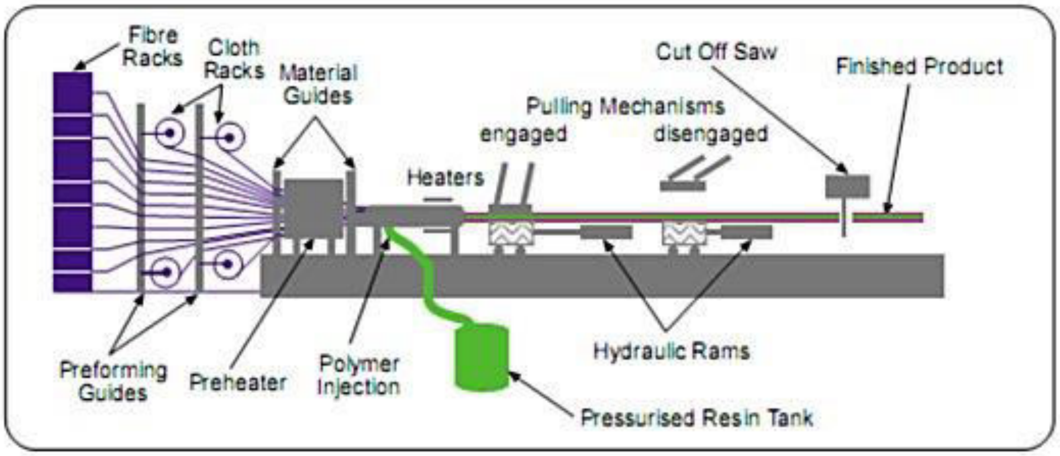
Advantages:
This can be a very fast and therefore economic method of laying material down.

- Resin content can be controlled by metering the resin onto each fibre tow through nips or dies.
- Fibre cost is minimised since there is no secondary process to convert fibre into fabric prior to use.
- Structural properties of laminates can be very good since straight fibres can be laid in a complex pattern to match the applied loads.

Disadvantages:

- The process is limited to convex shaped components.
- Fibre cannot easily be laid exactly along the length of a component.
- Mandrel costs for large components can be high.
- The external surface of the component is unmoulded, and therefore cosmetically unattractive.
- Low viscosity resins usually need to be used with their attendant lower mechanical and health and safety properties.

11 Pultrusion



Description:

Fibres are pulled from a creel through a resin bath and then on through a heated die. The die completes the impregnation of the fibre, controls the resin content and cures the material into its final shape as it passes through the die. This cured profile is then automatically cut to length. Fabrics may also be introduced into the die to provide fibre direction other than at 0°. Although pultrusion is a continuous process, producing a profile of constant cross-section, a variant known as 'pulforming' allows for some variation to be introduced into the cross-section. The process pulls the materials through the die for impregnation, and then clamps them in a mould for curing. This makes the process non-continuous, but accommodating of small changes in cross-section.

Material Options:

- Resins: Generally epoxy, polyester, vinylester and phenolic
- Fibres: Any
- Cores: Not generally used

Typical Applications:

Beams and girders used in roof structures, bridges, ladders, frameworks

Advantages:

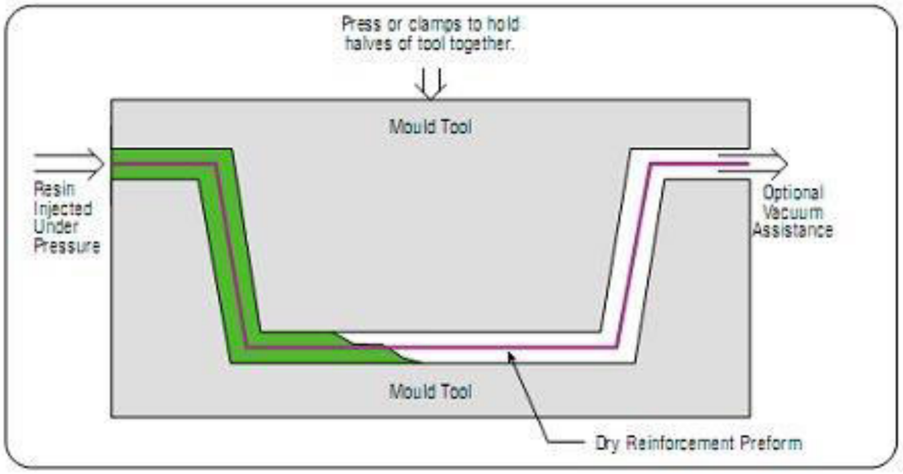
- This can be a very fast, and therefore economic, way of impregnating and curing materials.
- Resin content can be accurately controlled.
- Fibre cost is minimised since the majority is taken from a creel.
- Structural properties of laminates can be very good since the profiles have very straight fibres and high fibre volume fractions can be obtained.
- Resin impregnation area can be enclosed thus limiting volatile emissions.

Disadvantages:

- Limited to constant or near constant cross-section components.

- Heated die costs can be high.

Resin Transfer Moulding (RTM)



Description:

Fabrics are laid up as a dry stack of materials. These fabrics are sometimes pre-pressed to the mould shape, and held together by a binder. These 'preforms' are then more easily laid into the mould tool. A second mould tool is then clamped over the first, and resin is injected into the cavity. Vacuum can also be applied to the mould cavity to assist resin in being drawn into the fabrics. This is known as Vacuum Assisted Resin Injection (VARI). Once all the fabric is wet out, the resin inlets are closed, and the laminate is allowed to cure. Both injection and cure can take place at either ambient or elevated temperature.

Material Options:

- Resins: Generally epoxy, polyester, vinylester and phenolic, although high temperature resins such as bismaleimides can be used at elevated process temperatures.
- Fibres: Any. Stitched materials work well in this process since the gaps allow rapid resin transport. Some specially developed fabrics can assist with resin flow
- Cores: Not honeycombs, since cells would fill with resin, and pressures involved can crush some foams

Typical Applications:

Small complex aircraft and automotive components, train seats.

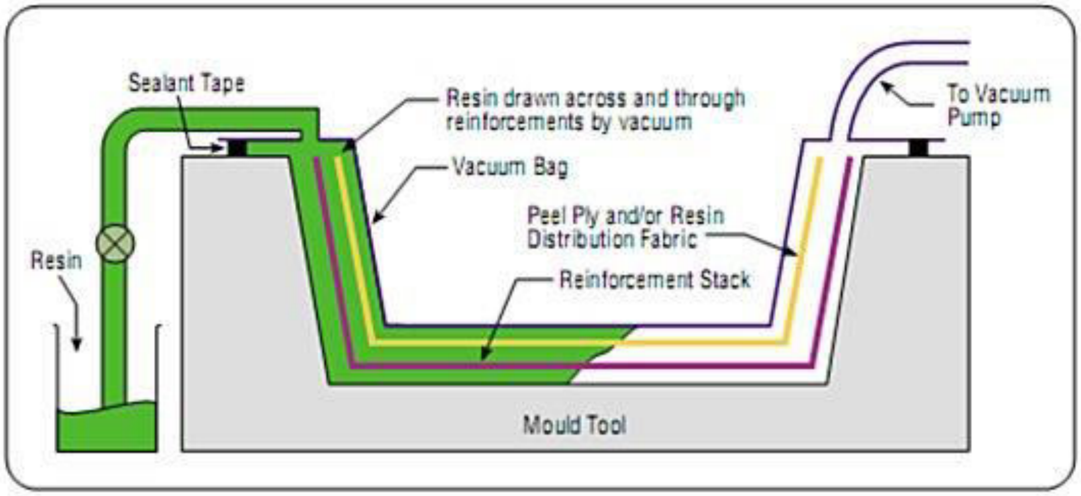
Advantages:

- High fibre volume laminates can be obtained with very low void contents.
- Good health and safety, and environmental control due to enclosure of resin.
- Possible labour reductions.
- Both sides of the component have a moulded surface.

Disadvantages:

- Matched tooling is expensive and heavy in order to withstand pressures.
- Generally limited to smaller components.
- Unimpregnated areas can occur resulting in very expensive scrap parts.

Other Infusion Processes



Description:

Fabrics are laid up as a dry stack of materials as in RTM. The fibre stack is then covered with peel ply and a knitted type of non-structural fabric. The whole dry stack is then vacuum bagged, and once bag leaks have been eliminated, resin is allowed to flow into the laminate. The resin distribution over the whole laminate is aided by resin flowing easily through the non-structural fabric, and wetting the fabric out from above.

Materials Options:

- Resins: Generally epoxy, polyester and vinylester.
- Fibres: Any conventional fabrics. Stitched materials work well in this process since the gaps allow rapid resin transport.
- Cores: Any except honeycombs.

Typical Applications:

Semi-production small yachts, train and truck body panels

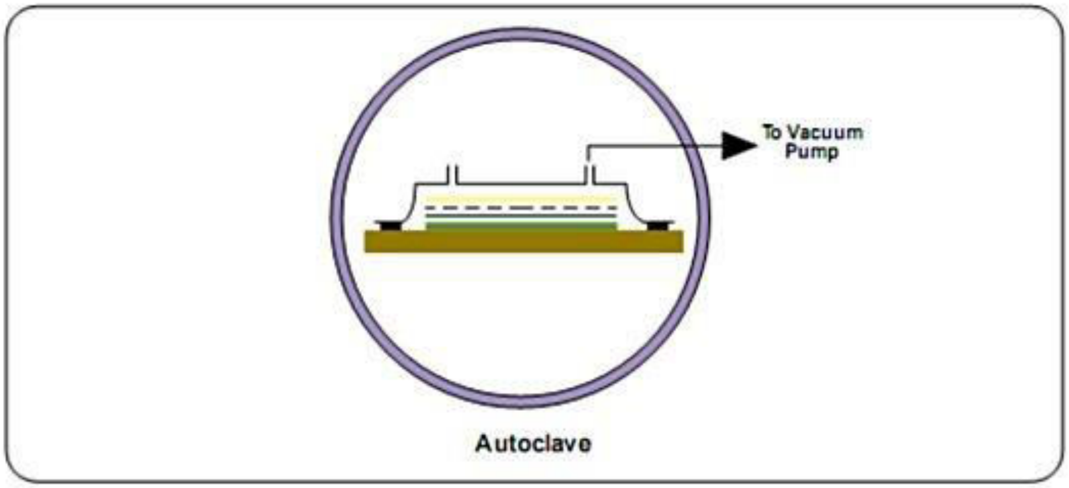
Advantages:

- As RTM above, except only one side of the component has a moulded finish.
- Much lower tooling cost due to one half of the tool being a vacuum bag, and less strength being required in the main tool.
- Large components can be fabricated.
- Standard wet lay-up tools may be able to be modified for this process.
- Cored structures can be produced in one operation.

Disadvantages:

- Relatively complex process to perform well.
- Resins must be very low in viscosity, thus comprising mechanical properties.

- Unimpregnated areas can occur resulting in very expensive scrap parts.



Description:

Fabrics and fibres are pre-impregnated by the materials manufacturer, under heat and pressure or with solvent, with a pre-catalyzed resin. The catalyst is largely latent at ambient temperatures giving the materials several weeks, or sometimes months, of useful life when defrosted. However to prolong storage life the materials are stored frozen. The resin is usually a near-solid at ambient temperatures, and so the pre-impregnated materials (prepregs) have a light sticky feel to them, such as that of adhesive tape. Unidirectional materials take fibre direct from a creel, and are held together by the resin alone. The prepregs are laid up by hand or machine onto a mould surface, vacuum bagged and then heated to typically 120-180°C. This allows the resin to initially reflow and eventually to cure. Additional pressure for the moulding is usually provided by an autoclave (effectively a pressurized oven) which can apply up to 5 atmospheres to the laminate.

Materials Options:

- Resins: Generally epoxy, polyester, phenolic and high temperature resins such as polyimides, cyanate esters and bismaleimides.

- Fibres: Any. Used either direct from a creel or as any type of fabric.
- Cores: Any, although special types of foam need to be used due to the elevated temperatures involved in the process.

Typical Applications:
Aircraft structural components (e.g. wings and tail sections), F1 racing cars, sporting goods such as tennis racquets and skis.

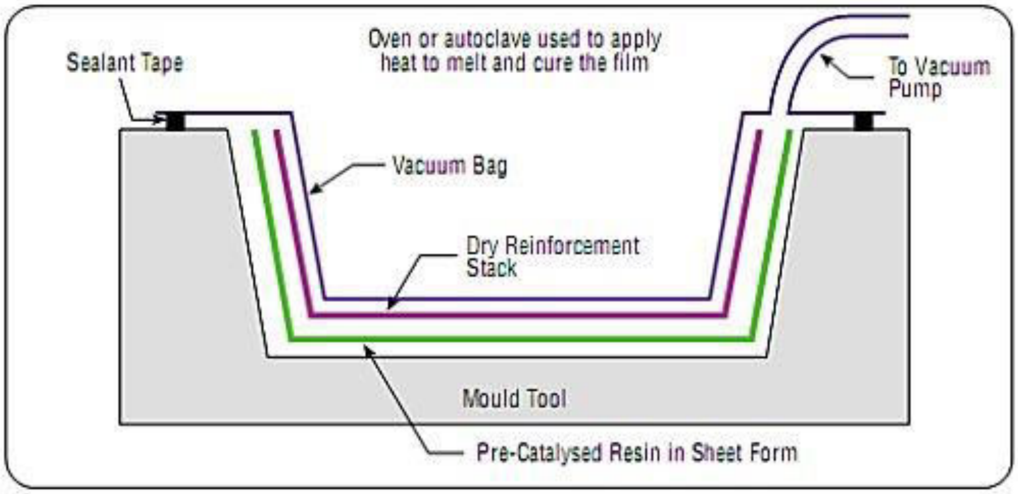
Advantages:

- Resin/catalyst levels and the resin content in the fibre are accurately set by the materials manufacturer. High fibre contents can be safely achieved.
- The materials have excellent health and safety characteristics and are clean to work with.
- Fibre cost is minimized in unidirectional tapes since there is no secondary process to convert fibre into fabric prior to use.
- Resin chemistry can be optimized for mechanical and thermal performance, with the high viscosity resins being impregnable due to the manufacturing process.
- The extended working times (of up to several months at room temperatures) means that structurally optimized, complex lay-ups can be readily achieved.
- Potential for automation and labour saving.

Disadvantages:

- Materials cost is higher for preimpregnated fabrics.
- Autoclaves are usually required to cure the component. These are expensive, slow to operate and limited in size.
- Tooling needs to be able to withstand the process temperatures involved.
- Core materials need to be able to withstand the process temperatures and pressures.

Resin Film Infusion (RFI)



Description:

Dry fabrics are laid up interleaved with layers of semi-solid resin film supplied on a release paper. The lay-up is vacuum bagged to remove air through the dry fabrics, and then heated to allow the resin to first melt and flow into the air-free fabrics, and then after a certain time, to cure.

Materials Options:

- Resins: Generally epoxy only.
- Fibres: Any
- Cores: Most, although PVC foam needs special procedures due to the elevated temperatures involved in the process

Typical Applications:

Aircraft radomes and submarine sonar domes.

Main Advantages:

- High fibre volumes can be accurately achieved with low void contents.
- Good health and safety and a clean lay-up, like prepreg.
- High resin mechanical properties due to solid state of initial polymer material and elevated temperature cure.
- Potentially lower cost than prepreg, with most of the advantages.
- Less likelihood of dry areas than SCRIMP process due to resin traveling through fabric thickness only.

Disadvantages:

- Not widely proven outside the aerospace industry.
- An oven and vacuum bagging system is required to cure the component as for prepreg, although the autoclave systems used by the aerospace industry are not always required.
- Tooling needs to be able to withstand the process temperatures of the resin film (which if using similar resin to those in low-temperature curing prepregs, is typically 60-100°C).
- Core materials need to be able to withstand the process temperatures and pressures.

UNIT-III

MECHANICAL PROPERTIES STIFNESS AND STRENGTH

Mechanical Properties Stiffness and Strength

Volume Fractions

Consider a composite consisting of fiber and matrix. Take the following symbol notations:

$v_{c,f,m}$ = volume of composite, fiber, and matrix, respectively
$\rho_{c,f,m}$ = density of composite, fiber, and matrix, respectively.

Now define the fiber volume fraction

Vf and the matrix volume fraction Vm as

$$V_f = \frac{v_f}{v_c}, \qquad V_m = \frac{v_m}{v_c}.$$

Note that the sum of volume fractions is

$$V_f + V_m = 1\,, \quad v_f + v_m = v_c.$$

Mass Fractions

Consider a composite consisting of fiber and matrix and take the following

symbol notation:

wc,f,m= mass of composite, fiber, and matrix, respectively. The mass fraction (weight fraction) of the fibers (Wf) and the matrix (Wm) are defined as

$$W_f = \frac{w_f}{w_c}, \text{ and}$$

$$W_m = \frac{w_m}{w_c}.$$

Note that the sum of mass fractions is

$$W_f + W_m = 1\,, \quad w_f + w_m = w_c\,.$$

From the definition of the density of a single material,

$$w_c = r_c v_c,$$

$$w_f = r_f v_f, \text{ and}$$

$$w_m = r_m v_m.$$

Substituting above equations, the mass fractions and volume fractions are related as

$$W_f = \frac{\rho_f}{\rho_c} V_f \text{, and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m,$$

in terms of the fiber and matrix volume fractions. In terms of individual constituent properties, the mass fractions and volume fractions are related by

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f,$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m .$$

One should always state the basis of calculating the fiber content of a composite. It is given in terms of mass or volume. Based on above Equation, it is evident that volume and mass fractions are not equal and that the mismatch between the mass and volume fractions increases as the ratio between the density of fiber and matrix differs from one.

Density

The derivation of the density of the composite in terms of volume fractions is found as follows. The mass of composite *wc* is the sum of the mass of the fibers *wf* and the mass of the matrix *wm* as

$$w_c = w_f + w_m.$$

$$\rho_c v_c = \rho_f v_f + \rho_m v_m, \quad \rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c} .$$

Using the definitions of fiber and matrix volume fractions from Equation

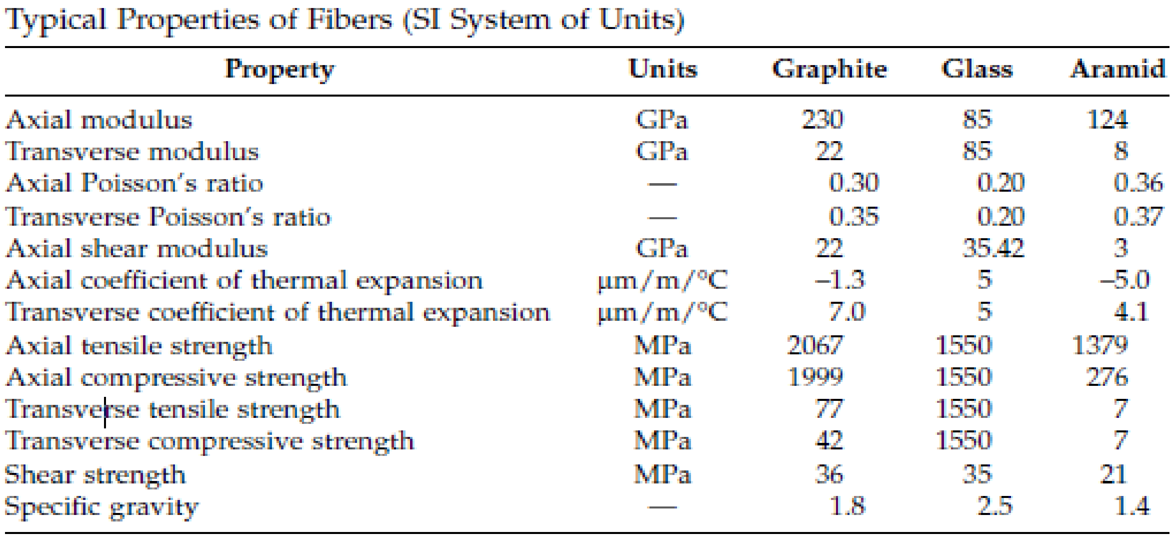
$$\rho_c = \rho_f V_f + \rho_m V_m.$$

Now, consider that the volume of a composite *vc* is the sum of the volumes of the fiber *vf* and matrix (*vm*):

$$v_c = v_f + v_m .$$

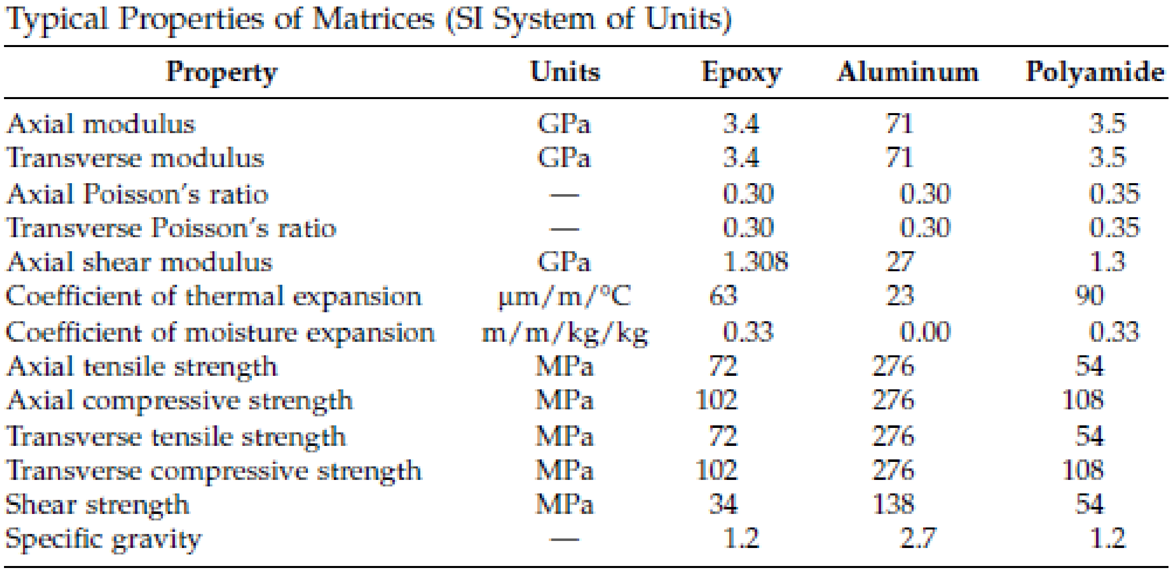
The density of the composite in terms of mass fractions can be found as

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}.$$



Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	µm/m/°C	–1.3	5	–5.0
Transverse coefficient of thermal expansion	µm/m/°C	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4



Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	µm/m/°C	63	23	90
Coefficient of moisture expansion	m/m/kg/kg	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

Idealization of Microstructure of Fibrous Composite:

As mentioned earlier, the micromechanics is a study at fibre and matrix level. Thus, the geometry of arrangement of the fibres and matrix in a composite is an essential requirement to develop a model for the study. Some of the methods do not use the geometry of arrangement. Most of the methods developed for micromechanical analysis assume that:

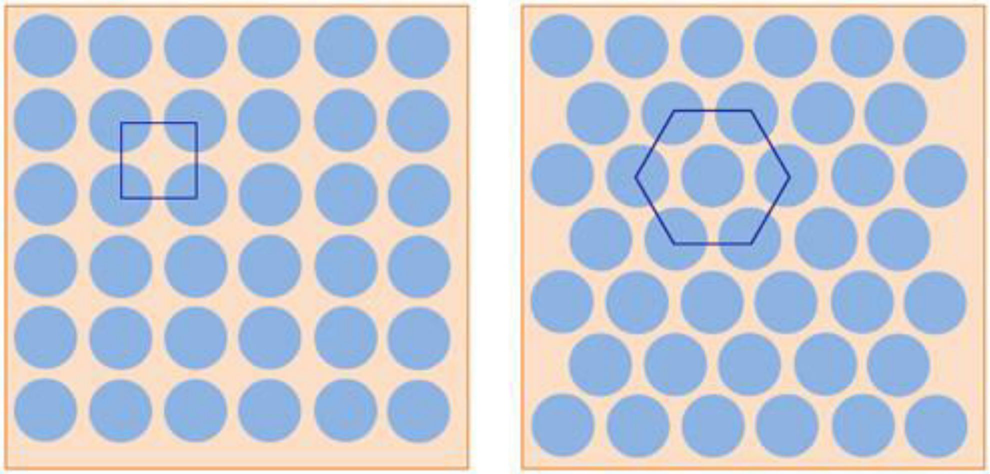
1. The fibers and matrix are perfectly bonded and there is no slip between them.

2. The fibres are continuous and parallel.
3. The fibres are assumed to be circular in cross section with a uniform diameter along its length.
4. The space between the fibres is uniform throughout the composite.
5. The elastic, thermal and hygral properties of fibre and matrix are known and uniform.
6. The fibres and matrix obey Hooke's law.
7. The fibres and the matrix are only two phases in the composite.
8. There are no voids in the composite.

There are many ways to idealize the cross section of a lamina. In Figure 1 are shown two popular idealizations. The most commonly preferred arrangements are square packed and hexagonal packed arrays of fibres in matrix. The square and hexagonal packed arrays can be as shown in Figure 1(a), and (b), respectively.

In these idealizations it is seen that due to symmetry and periodicity of these arrays one can consider only one array to analyze the lamina at micro scale. Further, if this one array represents the general arrangement of fibres with respect to matrix and the interactions of fibre and matrix phases, then such array is called ***Representative Volume Element* (RVE)**. Further, this RVE as a volume of material statistically represents a homogeneous material. In the analysis of an RVE the boundary conditions are chosen such that they reflect the periodicity. Thus, the arrays shown in Figure 1 are various RVEs. One should be able to see that the RVE also reflects the volume fractions. The term RVE was first coined by Hill in 1963.

For example, the square RVE represents a lower fibre volume fraction than a hexagonal RVE. Note that RVE is also called as ***Unit Cell.***



(a) Square packed array

(b) Hexagonal packed array

Figure 1.: Idealization of cross section of lamina

Strength of Material Approximations:

In general, the laminates made are thin. Hence, for such laminates the analysis done using Kirchhoff and plane stress assumptions is reasonably good. For such analysis, one needs the engineering constants that occur in defining planar constitutive equations. These engineering constants are:

1. E_1^* - **the axial modulus**
2. $E_2^* = E_3^*$ - **transverse modulus**
3. $\nu_{12}^* = \nu_{13}^*$ - **axial Poisson's ratio (for loading in** x_1**- direction)**
4. $G_{12}^* = G_{13}^*$- **axial shear modulus (shear stress parallel to the fibers)**

Further, it is seen that for transversely isotropic composite, four out of five (the fifth one is G_{23}^*) properties can be developed from this approach. For the planar hygro-thermal analysis of such laminates, one can also obtain the in-plane coefficients of thermal expansions α_1^* and α_2^* and hygroscopic expansion β_1^* and β_2^* as well.

It is important to note that this approach involves assumptions which do not necessarily satisfy the requirements of an exact elasticity solution. In this approach the effective properties will be expressed in terms of the elastic properties and volume fractions of the fiber and matrix. The lamina is considered to be an alternate arrangement of fibres and matrix. The RVE chosen in these derivations is shown in Figure 2. The RVE here does not take into account the cross sectional arrangement of fibres and matrix, rather it represents volume of the material through the cross sectional area of fibre and matrix.

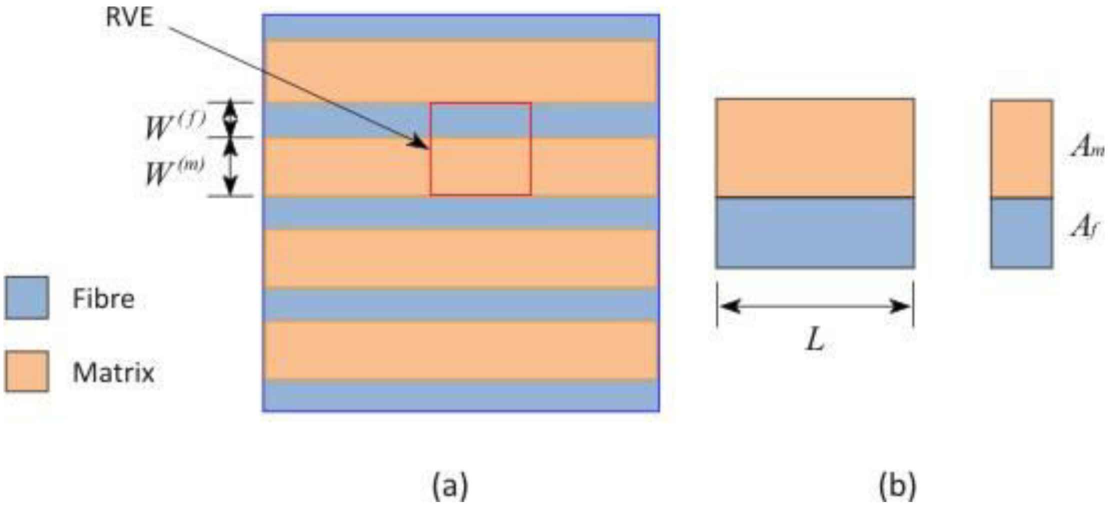


Figure 2: (a) Unidirectional lamina, (b) RVE for unidirectional composite for prediction of elastic properties

Let, A_f and A_m represent fibre area and matrix area, respectively. $W^{(f)}$ and $W^{(m)}$ represent fibre and matrix widths, respectively. L be the length of the RVE.

Effective Axial Modulus E_1^***:**

The unit cell as shown in Figure 2 is used to compute the effective axial modulus E_1^*. It should be noted that the thickness of the unit cell is not important in this computation. Further, the cross sectional shapes are not considered in this calculation. However, the cross sectional areas are important in this calculation. The thicknesses of the fibre and matrix constituents are same in the unit cell. Hence, the areas of the constituents represent the volume fractions of the constituents.

In the calculation of effective axial modulus, it is assumed that the axial strain in the composite is uniform such that the axial strains in the fibers and matrix are identical. This assumption is justified by the fact that the fibre and the matrix in the unit cell are perfectly bonded. Hence, the elongation in the axial direction of the fibre and matrix will also be identical. Thus, the strains in the fibre and matrix can be given as

$$\bar{\varepsilon}_1 = \varepsilon_1^{(f)} = \varepsilon_1^{(m)} = \frac{\Delta L}{L} \qquad (1)$$

where, $\bar{\varepsilon}_1$ is the axial strain in the composite and $\varepsilon_1^{(f)}$ and $\varepsilon_1^{(m)}$ are the axial strains in fibre and matrix, respectively. Now, let $E_1^{(f)}$ and $E^{(m)}$ be the axial Young's moduli of the fibre and matrix, respectively. We can give the axial stress in the fibre, $\sigma_1^{(f)}$ and matrix, $\sigma_1^{(m)}$ as

$$\sigma_1^{(f)} = E_1^{(f)}\varepsilon_1^{(f)} \quad \text{and} \quad \sigma_1^{(m)} = E^{(m)}\varepsilon_1^{(m)}$$

Using the above equation and the cross section areas of the respective constituent in the unit cell, we can calculate the forces in them as

$$F_1^{(f)} = \sigma_1^{(f)}A_f \quad \text{and} \quad F_1^{(m)} = \sigma_1^{(m)}A_m$$

The total axial force in the composite is sum of the axial forces in fibre and matrix. Thus, the total axial force in the composite substituting the expressions for axial strains in fibre and matrix from Equation (1) in above equation, can be given as

$$F_1 = F_1^{(f)} + F_1^{(m)} = \sigma_1^{(f)}A_f + \sigma_1^{(m)}A_m = \left(E_1^{(f)}A_f + E^{(m)}A_m\right)\frac{\Delta L}{L} \quad (2)$$

Now $\bar{\sigma}_1$ be the average axial stress in composite. The total cross sectional area of the composite is $A = A_f + A_m$. Thus, using the average axial stress and cross sectional area of the composite, the axial force is

$$F_1 = \bar{\sigma}_1 A \qquad (3)$$

Thus, combining Equation (2) and Equation (3) and rearranging, we get

$$\bar{\sigma}_1 = \left(E_1^{(f)} \frac{A_f}{A} + E^{(m)} \frac{A_m}{A} \right) \frac{\Delta L}{L} \qquad (4)$$

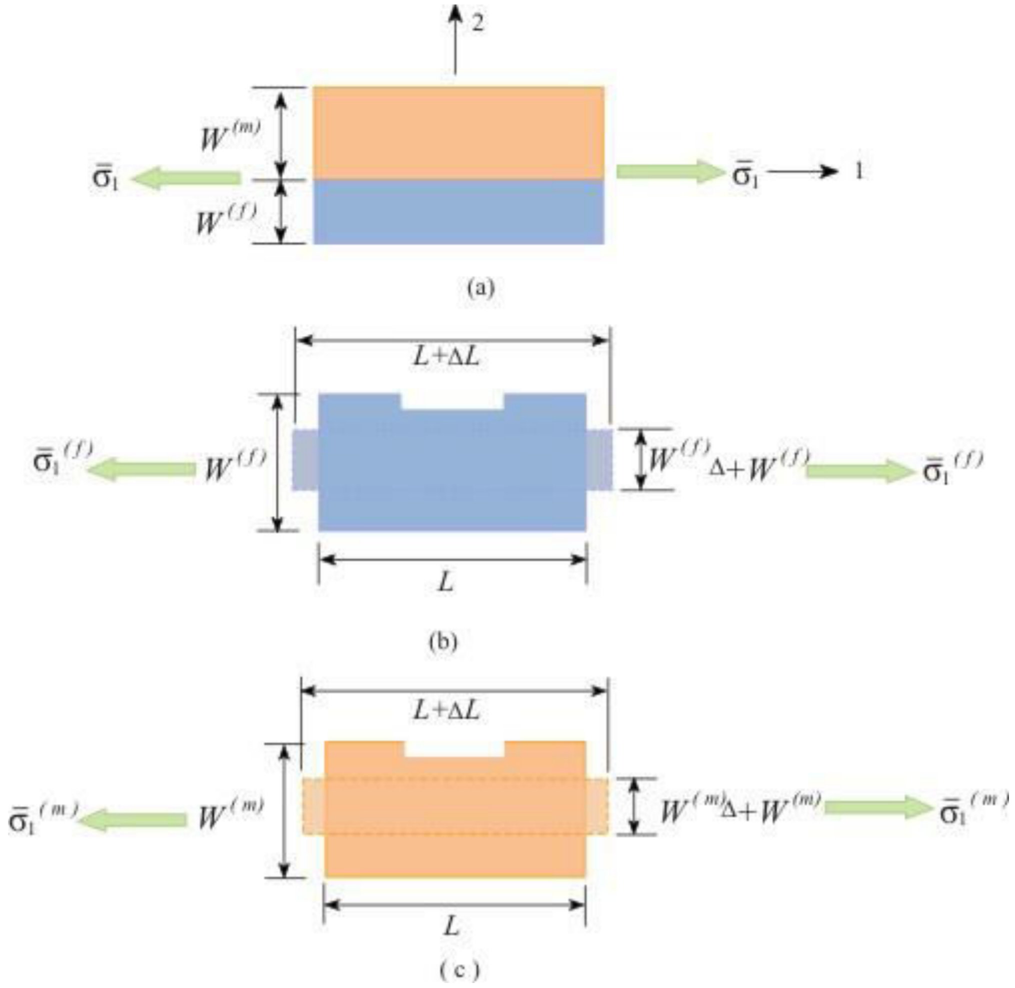


Figure 3: (a) Undeformed unit cell under $\bar{\sigma}_1$ (b) and (c) deformed individual constituents of the unit cell

Let us define

$$\bar{\sigma}_1 = E_1^{*} \bar{\varepsilon}_1 = E_1^{*} \frac{\Delta L}{L} \qquad (5)$$

Further, noting that the ratios $\frac{A_f}{A}$ and $\frac{A_m}{A}$ for same length of fibre and matrix represent the fibre and matrix volume fractions, respectively. Thus, combining Equations (4) and (5), we get

$$E_1^* = E_1^{(f)}V_f + E^{(m)}V_m = E_1^{(f)}V_f + E^{(m)}\left(1 - V_f\right) \quad (6)$$

The above equation relates the axial modulus of the composite to the axial moduli of the fibre and matrix through their volume fractions. Thus, the effective axial modulus is a linear function of the fiber volume fraction. This equation is known as rule of mixtures equation. It should be noted that the effective properties are functions of the fiber volume fractions; hence it should always be quoted in reporting the effective properties of a composite.

Effective Axial (Major) Poison's Ratio ν_{12}^***:**

To determine the effective axial Poisson's ratio we consider the loading as in the case applied for determining the effective axial modulus. Here, for this loading we have $\bar{\sigma}_1 \neq 0$ and other stresses are zero. We define the effective axial Poisson's ratio as

$$\nu_{12}^* = -\frac{\bar{\varepsilon}_2}{\bar{\varepsilon}_1}$$

The effective strain in direction 2 from Figure 3(b) and (c) can be given as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W^{(f)} + W^{(m)}}$$

Now, the changes in $W^{(f)}$ and $W^{(m)}$ can be obtained using the Poisson's ratio of individual constituents. The axial Poisson's ratios for fibre and matrix are given as

$$\nu_{12}^{(f)} = -\frac{\varepsilon_2^{(f)}}{\varepsilon_1^{(f)}} = -\frac{\Delta W^{(f)} / W^{(f)}}{\Delta L / L} \quad \text{and} \quad \nu^{(m)} = -\frac{\varepsilon_2^{(m)}}{\varepsilon_1^{(m)}} = -\frac{\Delta W^{(m)} / W^{(m)}}{\Delta L / L} \quad (7.24)$$

Thus, the changes in $W^{(f)}$ and $W^{(m)}$ are given as

$$\Delta W^{(f)} = -\nu_{12}^{(f)} W^{(f)} \frac{\Delta L}{L} \quad \text{and} \quad \Delta W^{(m)} = -\nu^{(m)} W^{(m)} \frac{\Delta L}{L} \quad (7.25)$$

The total change in W is given as

$$\Delta W = \Delta W^{(f)} + \Delta W^{(m)} \qquad (7)$$

The strain in direction 2 for the composite can be given using Equation (6) and Equation (7) as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W} = -\left(\nu_{12}^{(f)} \frac{W^{(f)}}{W} + \nu^{(m)} \frac{W^{(m)}}{W}\right) \frac{\Delta L}{L} \quad (8)$$

Here, $\frac{W^{(f)}}{W}$ and $\frac{W^{(m)}}{W}$ denote the fibre and matrix volume fractions for same length of fibre and matrix. Note that $\frac{\Delta L}{L}$ denotes the effective axial strain $\bar{\varepsilon}_1$. Thus, from Eq. (8) the effective axial Poisson's ratio is written as

$$\nu_{12}^{(*)} = \nu_{12}^{(f)} V_f + \nu^{(m)} V_m$$

The above equation is the rule of mixtures expression for composite axial Poisson's ratio.

Effective Transverse Modulus E_2^*:

Here, we are going to derive the effective transverse modulus by loading the RVE in direction 2 as shown in Figure 4(a). There are two considerations while deriving this effective modulus. The first approach considers that the deformation of the each constituent is independent of each other as shown in Figure 4(b) and (c) and the deformation in direction 1 is not considered. The second approach considers that deformations of the fibre and matrix in direction 1 are identical as they are perfectly bonded.

To calculate the effective modulus in direction 2, a stress $\bar{\sigma}_2$ is applied to the RVE as shown in Figure 4(a).

First Approach:

As mentioned, the fibre and matrix deform independently of each other. The resulting deformation in direction 1 is not considered here. This assumption is simplistic and was used by early researchers.

The fibre and matrix are subjected to same state of stress. The state of stress is unidirectional, that is, $\sigma_2^{(f)} = \sigma_2^{(m)} = \bar{\sigma}_2$. Now, using the individual moduli and deformations in direction 2, these stresses can be given as

$$\sigma_2^{(f)} = E_2^{(f)} \varepsilon_2^{(f)} = E_2^{(f)} \frac{\Delta W^{(f)}}{W^{(f)}}$$

$$\sigma_2^{(m)} = E^{(m)} \varepsilon_2^{(m)} = E^{(m)} \frac{\Delta W^{(m)}}{W^{(m)}}$$

From this equation we can write the individual deformations, which give the total deformation in direction 2 as

$$\Delta W = \Delta W^{(f)} + \Delta W^{(m)} = \left(\frac{W^{(f)}}{E_2^{(f)}} + \frac{W^{(m)}}{E^{(m)}} \right) \bar{\sigma}_2$$

Now, the composite strain in direction 2 can be calculated from the definition as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \left(\frac{W^{(f)}}{W} \frac{1}{E_2^{(f)}} + \frac{W^{(m)}}{W} \frac{1}{E^{(m)}} \right) \bar{\sigma}_2$$

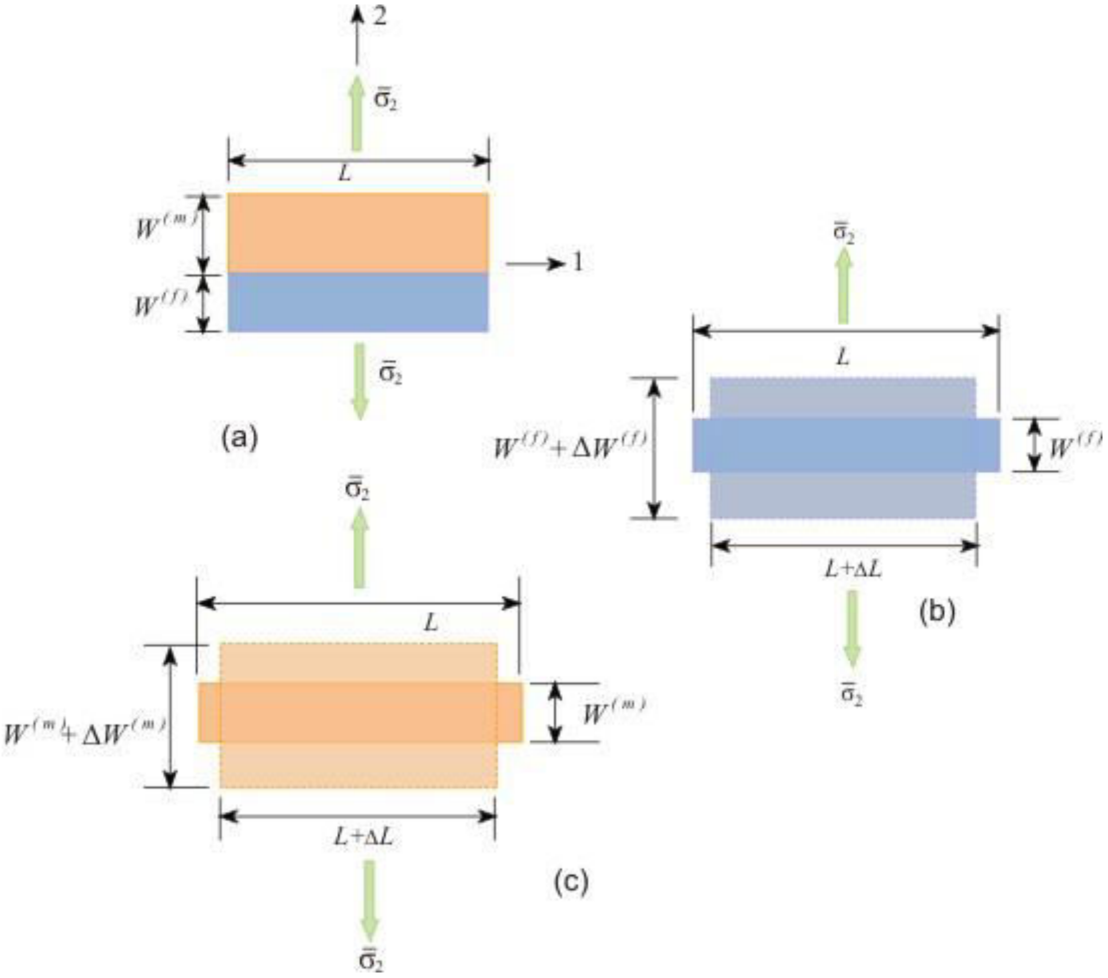


Figure 4: (a) Undeformed unit cell under uniform $\bar{\sigma}_2$ stress (b) and (c) deformed individual constituents of the unit cell

Introducing the volume fractions in the above equation,

$$\bar{\varepsilon}_2 = \left(V_f \frac{1}{E_2^{(f)}} + V_m \frac{1}{E^{(m)}} \right) \bar{\sigma}_2$$

Noting that $\frac{\bar{\sigma}_2}{\bar{\varepsilon}_2} = E_2^*$, from the above equation, we get

$$\frac{1}{E_2^*} = \frac{V_f}{E_2^{(f)}} + \frac{V_m}{E^{(m)}} = \frac{V_f}{E_2^{(f)}} + \frac{(1-V_f)}{E^{(m)}}$$

This equation is the rule of mixtures equation for effective modulus E_2^*.

Background to Mechanical Testing of Composites:

Objectives of Mechanical Testing:

The development of the mechanical testing of the materials depends upon other scientific factors. These factors help in better understanding and facilitate the progress in evaluating the various processes. These processes include:

1. quality control of a process
2. quality assurance for the material developed and structure fabricated from thereof
3. better material selection
4. comparisons between available materials
5. can be used as indicators in materials development programmes
6. design analysis
7. predictions of performance under conditions other than test conditions
8. starting points in the formulation of new theories

It should be noted that these processes are dependent upon each other. However, if they are considered individually then the data required can be different for the evaluation. For example, some tests are carried out as multipurpose tests using various processes. A conventional tensile test carried out under fixed conditions may serve quality control function whereas one carried out varying factors like temperature, strain rate, humidity etc. may provide information on load bearing capacity of the material.

The properties evaluated for materials like composite is very sensitive to various internal structure factors. However, these factors depend mainly upon the fabrication process or other factors. The internal structure factors that affect the properties are, in general, at atomic or molecular level. These factors mostly affect the matrix and fibre-matrix interface structure.

The mechanical properties of the fibrous composite depend on several factors of the composition. These factors are listed below again for the sake of completeness.

1. properties of the fibre
2. surface character of the fibre
3. properties of the matrix material

4. properties of any other phase
5. volume fraction of the second phase (and of any other phase)
6. spatial distribution and alignment of the second phase (including fabric weave)
7. nature of the interfaces

Another important factor is processing of the composites. There are many parameters that control the processing of composites that access the quality of adhesion between fibre and matrix, physical integrity and the overall quality of the final structure.

In case of composite the spatial distribution and alignment of fibres are the most dominating factor which causes the variation of properties. The spatial distribution and alignment of the fibres can change during the same fabrication process. Thus, for a given fabrication process the property evaluated from the composite material may show a large variation.

Tensile Testing

The well known purpose of the tensile testing is to measure the ultimate tensile strength and modulus of the composite. However, one can measure the axial Poison's ratio with additional instrumentations. The standard specimen used for tensile testing of continuous fiber composites is a flat, straight-sided coupon. A flat coupons in ASTM standard D 3039/D 3039M-93 for 0° and 90° have been shown.

The specimen, as mentioned above is flat rectangular coupon. The tabs are recommended for gripping the specimen. It protects the specimen from load being directly applied to the specimen causing the damage. Thus, the load is applied to the specimen through the grips. Further, it protects the outer fibres of the materials. The tabs can be fabricated from a variety of materials, including fiberglass, copper, aluminum or the material and laminate being tested. When the tabs of composite material are used then according to ASTM specifications the inner plies of the tabs should match with the outer plies of the composite. This avoids the unwanted shear stresses at the interface of the specimen and tabs. However, the recent versions of the ASTM standards allow the use of tabs with reinforcement at $\pm 45^\circ$. Further, end-tabs can also facilitate accurate alignment of the specimen in the test machine, provided that they are symmetrical and properly positioned on the specimen. The tabs are pasted to the specimen firmly with adhesive.

This specimen can provide data on:

1. The axial modulus E_x,
2. In-plane and through thickness Poisson's ratio γ_{xy}, γ_{xz}
3. Tensile ultimate stress σ_x^{ult},

4. Tensile ultimate strain ε_x^{ult} ,
5. Any nonlinear, inelastic response

In general, the tensile tests are done on coupons with 0° laminae/laminate for corresponding axial properties and coupons with 90° laminae/laminate for corresponding transverse properties. The off axis laminae specimen also provides data on coefficient of mutual influence and the in-plane shear response.

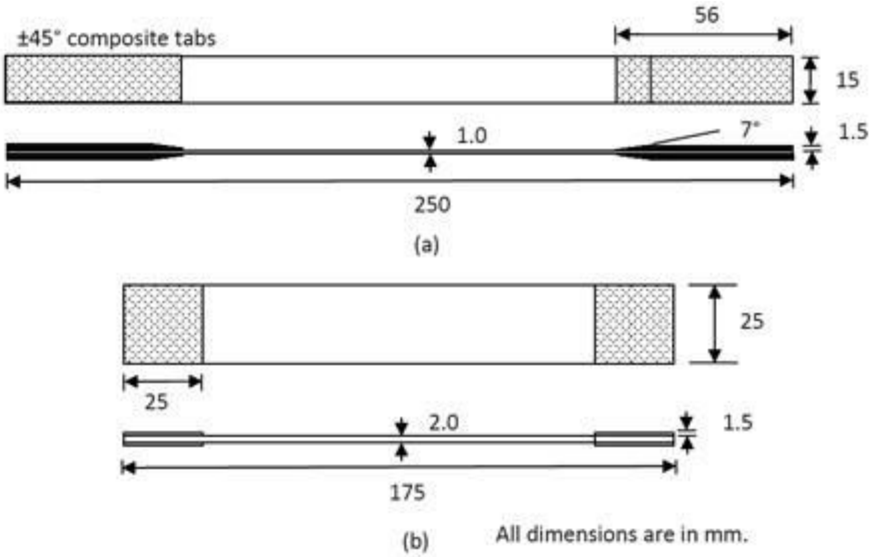


Figure 4:Composite tensile test specimens
(a) ASTM D 3039 for 0° and
(b) ASTM D 3039 for 90°.

***Orthotropic Laminae and Laminate*:**

For orthotropic, symmetric laminates with 0° and 90° laminae, the effective axial modulus and Poisson's ratio is given as

$$E_x = \frac{1}{a_{11}^*}, v_{xv} = \frac{-a_{12}^*}{a_{11}^*}$$

where, the quantities with asterisk are for laminate as mentioned in Chapter on Laminate Theory. These properties can be measured directly from a tensile test on a specimen of thickness t under axial force per unit length N_x as follows:

$$E_x = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{N_{xx}}{t\varepsilon_{xx}}, v_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}$$

The tensile strength is defined as the average stress at failure. Thus, the tensile strength can be given using the maximum applied force per unit length N_x and thickness t as

$$\bar{\sigma}_x^{ult} = \frac{N_x^{\max}}{t}$$

It should be noted that the failure of laminates is often influenced by inter laminar stresses along the free edge effects of the coupon. These factors will be explained in brief in one of the lecture.

The measurement of tensile strength by experiments can also provide information on the comparison of laminate theory with experiments.

Off-Axis Laminae

One can measure the tensile properties by conducting experiments on off-axis laminae. However, there are certain issues associated with this kind of experiments. For example, the presence of axial-shear coupling is associated with the nonzero a_{16}^* . Alternately, one can say that this term is associated with coefficient of mutual influence $\eta_{xy,x}$. Hence, these tests are not straight forward as in case of symmetric laminates with 0° and 90° laminae. Therefore, sometimes these tests are called as specialized tests.

When the experiments are conducted to measure the properties like E_x, v_{xy} and v_{xz} one can get the other properties along with these tests. For example, the coefficient of mutual influence $\eta_{xy,x}$, the nonlinear response and strength of an off-axis lamina for given fibre orientation can also be obtained.

There is an important issue associated with these tests is that what boundary conditions one should impose on the specimen? If a pure, uniform state of axial stress $\sigma_{xx} \neq 0, \sigma_{yy} = \tau_{xy} = 0$ can be applied to the ends and sides of a specimen and the specimen is free to assume any desired deformation pattern, the state of stress will be uniform and constant through-out the specimen. The deformation pattern is shown in Figure 8.4(a).

For uniform, far-field axial stress loading, that is $\sigma_{xx} \neq 0$, the stresses in principal material directions can be given as

$$\sigma_{11} = m^2\sigma_{xx}, \sigma_{22} = n^2\sigma_{xx}, \tau_{12} = -mn\sigma_{xx}$$

Further, the global elastic constants associated with axial stress loading are measured as

$$E_x = \frac{\sigma_{xx}}{\varepsilon_{xx}}, v_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}, \eta_{xy,x} = \frac{\gamma_{xy}}{\varepsilon_{xx}}$$

Combining above two equations, we get

$$\varepsilon_{11} = \frac{\sigma_{11}}{E_1} - \frac{v_{21}\sigma_{22}}{E_2} = (m^2 - n^2 v_{12})\frac{\sigma_{xx}}{E_1}$$

$$\varepsilon_{22} = -\frac{v_{12}\ \sigma_{11}}{E_1} + \frac{\sigma_{22}}{E_2} = \left(-\frac{v_{12}m^2}{E_1} + \frac{n^2}{E_2}\right)\sigma_{xx}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}} = \frac{-mn\ \sigma_{xx}}{G_{12}}$$

From the above equation all three strain components can be obtained for non zero value of axial stress. Thus, from the third of the above equation we can find the shear modulus.

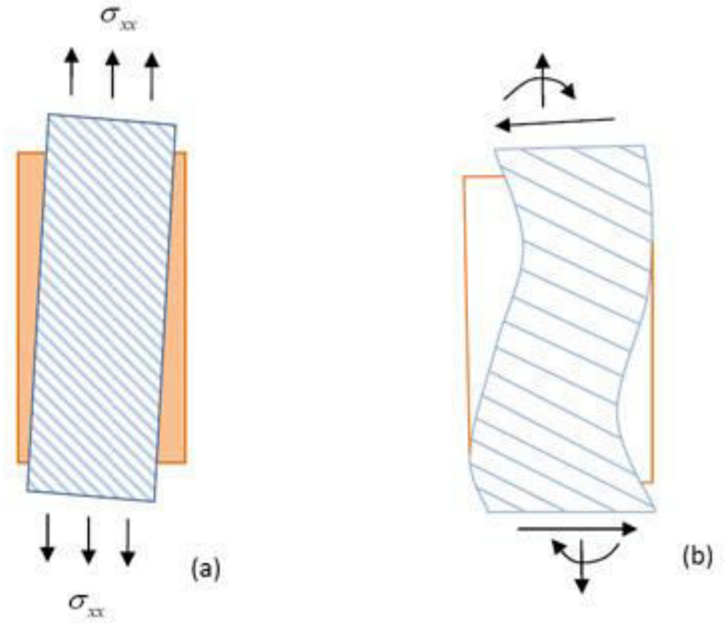


Figure 5: Axial load on off-axis laminae with effect of end constraint
(a) unconstrained displacement and
(b) constrained displacement

It is not easy to apply pure, uniform tensile stress to an off-axis coupon. The specimens are gripped in such a manner that the ends of the specimen are constrained and boundary condition is actually a specification of the axial end displacement. Further, there are more issues with these tests like the constrained displacement induces a doubly curved displacement field in the specimen. The deformed shape of the coupon with restrictions on the ends is depicted in Figure 5(b). We will not deal the complete analysis for the measurements of the properties with tests on off-axis laminae.

The bone shaped specimens for chopper-fiber, metal matrix composite tensile tests. More details can be seen in ASTM D3552-77(1989). Further, for the tensile testing for transverse properties of hoop-wound polymer matrix composite cylinders are used. The details of this testing can be seen in ASTM D5450/D5450M-93.

Measurement of modulus

It should be noted that due to progressive damage the stiffness of the lamina or laminae/laminate changes causing the stress strain curve to be non-linear. The measurement of modulus in a tensile testing from a non-linear loading curve can be done by three methods.

In the first method the modulus is taken as a tangent to the initial part of the curve. In the second method a tangent is constructed at a specified strain level. For example, in the Figure 5 the modulus is measured at 0.25% strain or 0.0025 strain (Point B). In the third method, a secant is constructed between two points. For example in Figure 8.5 a secant is constructed between points A and B. Typically, the strain values at these points are 0.0005 and 0.0025. In ASTM standards the secant is called as *chord*. The modulus measured by these methods is known as ***'initial tangent modulus'***, ***'B% modulus'* and *'A%-B% secant (chord) modulus'***, respectively.

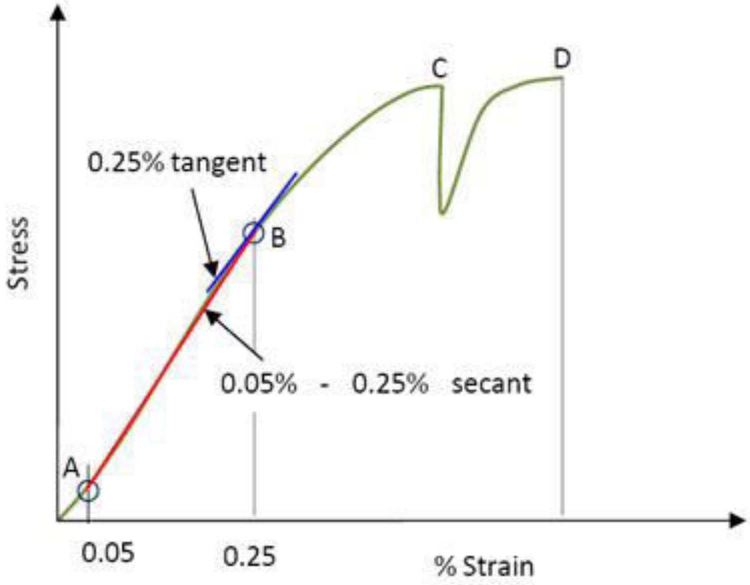


Figure 6: Typical tensile stress-strain curve with details

Compression Testing

Most of the structural members include the compression members. Such members can be loaded directly in compression or under a combination of flexural and compression loading. The axial stiffness of such members depends upon the cross-sectional area. Thus, it is proportional to the weight of the structure. One can alter the stiffness by changing the geometry of the cross section within limits. However, some of the composites have low compressive strength and this fact limits the full potential application of these composites.

The compression testing of the composites is very challenging due to various reasons. The application of compressive load on the cross section can be done in three ways: directly apply the compressive load on the ends of a specimen, loading the edges in shear and mixed shear and direct loading. These three ways of imposing the loads for compression testing are shown .

During compression loading the buckling of the specimen should be avoided. This demands a special requirement on the holding of the specimen for loading purpose. Further, it demands for special geometry of the specimen. These specimens are smaller in size as compared to the tensile testing specimens. A compression test specimen according to ASTM D695 (modified) standard is shown.

The compression testing of composites is a vast topic. Additional reading on this topic from other literature is suggested to readers.

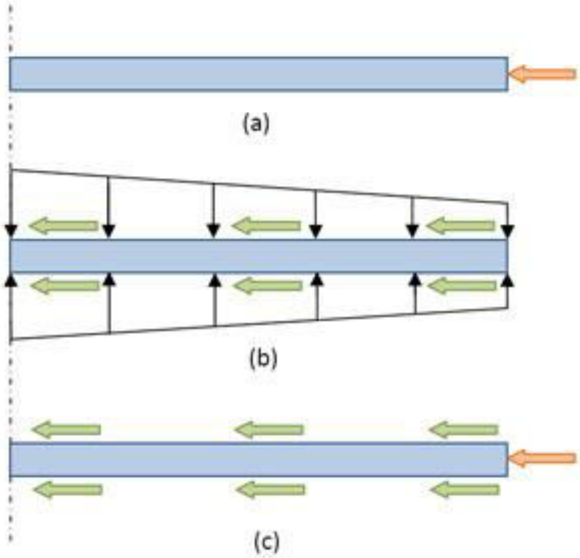


Figure 7: Load imposition methods for compression testing. (a) Direct end loading (b) Shear loading and (c) Mixed shear and direct loading

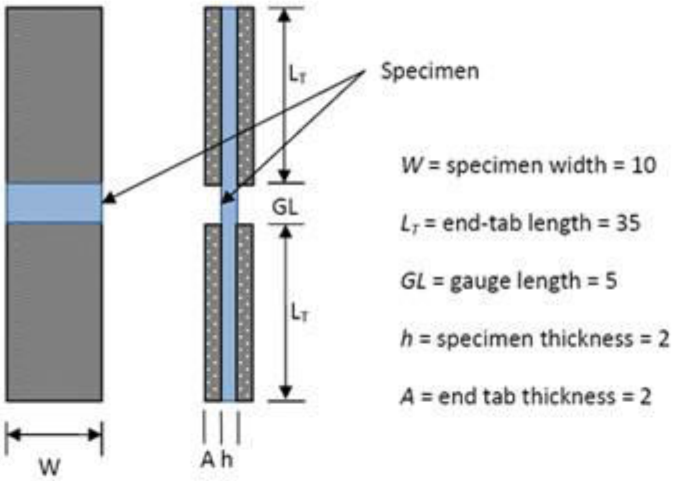


Figure 8: Composite compression test specimen according to ASTM D695 (modified) standard.

Shear Testing

Here we will see measurement of in-plane shear modulus G_{12} only. The methods are listed below:

1. Tension of a $[\pm 45]_s$ laminate
2. Tension of an off-axis lamina
3. Torsion of a unidirectional tube
4. Iosipescu shear of unidirectional laminae and cross ply laminates
5. Rail shear of unidirectional laminae
6. Picture frame test

1. $[\pm 45]_s$***Tensile Test*****:**

A tension test on $[\pm 45]_s$ laminate is popularly used test for the measurement of in-plane shear modulus G_{12}. The more details of this test are available in ATSM standard D3518/D3518/M–91. According to ASTM standard the method uses a 250 mm long rectangular specimen with width 25 mm and thickness 2 mm. Further, it is recommended that for materials constructed with layers thicker than 0.125mm, the laminate should consist of 16 layers, that is, $[\pm 45]_{4s}$.The specimen is shown . The dimensions in this figure are in mm.

When a $[\pm 45]_{4s}$ is subjected to axial tensile stress $\bar{\sigma}_{xx}$ then stresses in principal material coordinates developed in each of the $+45^\circ$ and -45° lamina are given as

$$\begin{aligned}\sigma_{11} &= B\bar{\sigma}_{xx}\\ \sigma_{22} &= (1-B)\bar{\sigma}_{xx}\\ \tau_{12} &= \frac{-1}{2mn}[B(1-2m^2)+m^2]\bar{\sigma}_{xx}\end{aligned}$$

where,

$$B = \left[\frac{m^2(2m^2-1)+4m^2n^2\frac{G_{12}}{E_2}\left(\frac{E_2}{E_1}v_{12}+1\right)}{4m^2n^2\frac{G_{12}}{E_2}\left(\frac{E_2}{E_1}+2\frac{E_2}{E_1}v_{12}+1\right)+(2m^2-1)(m^2-n^2)}\right]$$

and other quantities as defined in earlier chapters. For a special case with $\theta = 45^{\circ}$ we get the shear stress as

$$\tau_{12} = (\pm\theta) = \mp\frac{\bar{\sigma}_{xx}}{2}$$

Thus, from this equation one can see that the shear stress in principal material directions is statically determinate, that is, it is independent of material properties of the specimen and only depends upon the magnitude of the applied stress. The magnitude of this stress is half of the applied stress.

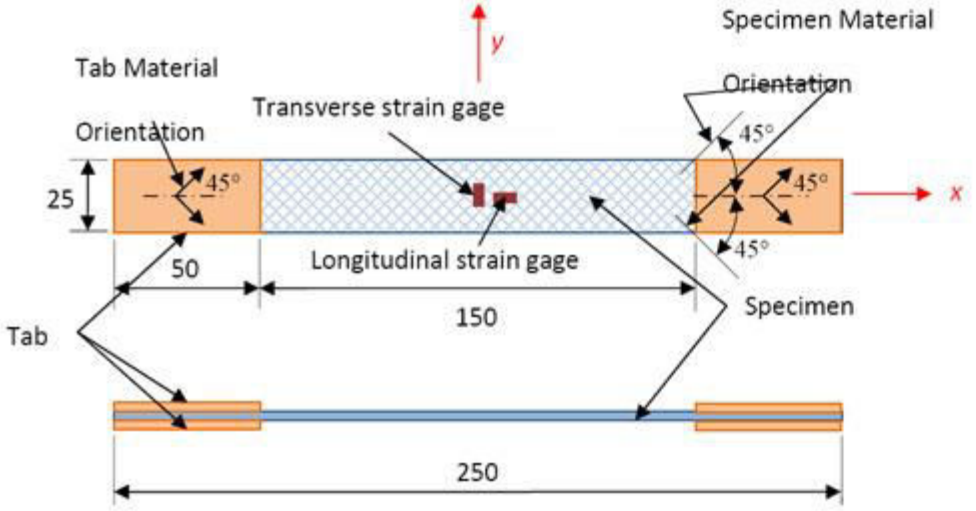


Figure 9: Specimen geometry and strain gage positioning for $[\pm 45]_{4s}$ tensile testing

From the knowledge of linear elastic behaviour of the orthotropic materials it is clear that the shear response is uncoupled from the normal response. Hence, in-plane shear modulus G_{12} can be determined directly from a tensile test on a $[\pm 45]_{4s}$ laminate.

Now the shear strain γ_{12} in principal material coordinates can be found by transformation of the measured axial and transverse strains ε_{xx} and ε_{yy}. It should be noted that the shear strain γ_{xy} is zero for orthotropic laminates under tension and γ_{12} is independent of γ_{xy} for $\theta = \pm 45^{\circ}$ (see the strain transformation relations). Thus, from the strain transformation relations, we can get the shear strain in principal material directions as

$$\gamma_{12} = -\left(\varepsilon_{xx} - \varepsilon_{yy}\right)$$

Thus, from the definition of the shear modulus we get

$$G_{12} = \frac{\bar{\sigma}_{xx}}{2\left(\varepsilon_{xx} - \varepsilon_{yy}\right)}$$

The above equation can be rearranged in the following manner to express the shear modulus in terms of effective properties of $[\pm 45]_{4s}$ laminate.

$$G_{12} = \frac{\dfrac{\bar{\sigma}_{xx}}{\varepsilon_{xx}}}{2\left(\dfrac{\varepsilon_{xx}}{\varepsilon_{xx}} - \dfrac{\varepsilon_{yy}}{\varepsilon_{xx}}\right)} = \frac{E_x}{2\left(1 + v_{xy}\right)}$$

Here, E_x is the effective modulus of the $[\pm 45]_{4s}$ laminate.

The measurement of in-plane shear modulus from shear stress-strain curve is done as follows. The shear stress-strain curve for $\pm 45^{\circ}$ specimen is obtained first. A typical shear stress-strain curve for such a specimen is shown. The shear modulus is obtained from the initial slope of the this curve in the range of 0.1-0.5% strain as

$$G_{12} = \frac{\tau'_{12} - \tau'_{12}}{\gamma'_{12} - \gamma'_{12}}$$

The tensile test on $\pm 45^{\circ}$ specimen provides an acceptable method for the measurement of in-plane shear modulus. However, one should be careful while interpreting the ultimate shear strength and strain. It should be noted that the laminae are subjected to a biaxial state of stress and not a pure shear. The normal stresses act along the shear planes causing the onset of mixed mode fracture. Other kind of failure like multiple ply cracking, fibre rotation and edge or internal delaminations occur prior to final failure. Therefore, the true failure is very difficult to determine. The shear strength is specified by different standards corresponding either to the ultimate load generated during the test or to a specified strain level. It is recommended in ISO standard that the test be terminated at $\gamma_{12} = 5\%$. The shear strength is taken as the peak load at or before 5% strain.

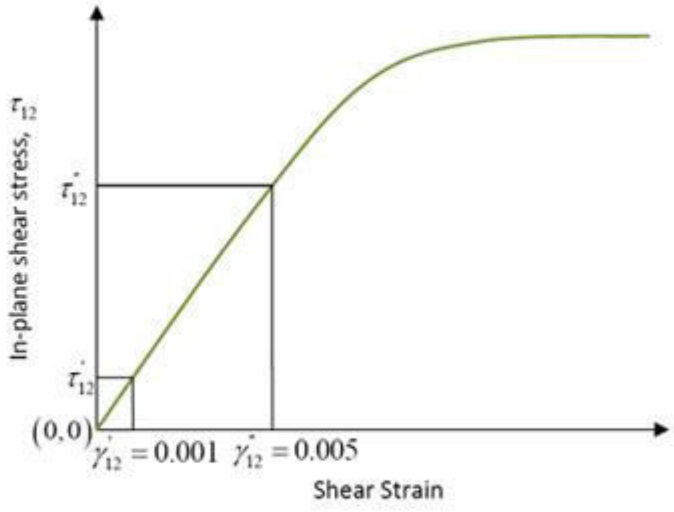


Figure 10: Typical shear stress-strain curve for specimen

***2. Shear Of an Off-Axis Lamina*:**

In similar way to the tensile testing of a $[\pm 45]_{4s}$ laminate one can use a unidirectional off-axis tensile coupon to determine the shear response of a composite in the principal material coordinates. A tensile test on 10° off-axis lamina is a commonly used. Specimen has same geometry . The state of stress in principal material coordinate directions can be obtained from transformation relations. Since, the shear response in the principal material coordinates is uncoupled from the normal response we can write the shear modulus as

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

The shear stress in the principal material directions due to axial tensile stress can be given using transformation relations as

$$\bar{\tau}_{12} = -mn\bar{\sigma}_{xx}$$

The shear strain is measured from the strains $\varepsilon_{xx}, \varepsilon_{yy}$ and γ_{xy} with the help of strain transformation relations. Then the apparent shear modulus $\bar{G}_{12}$ can be given as

$$\bar{G}_{12} = \frac{-mn\bar{\sigma}_{xx}}{\gamma_{12}}$$

1. Rail Shear Test:

This is a very popular method used to measure in-plane shear properties. This method is extensively used in aerospace industry. The shear loads are imposed on the edges of the laminate using specialized fixtures. There are two types of such fixtures: Two rail and three rail fixture. The ASTM D4255 standard covers the specification for two and three rail specimens for both continuous and discontinuous (0° and 90° fibre alignment), symmetric laminates and randomly oriented fibrous laminates.

a. Two Rail Shear Test

The two rail shear test fixture along with a laminate to be tested is shown . The Figure shows the specimen geometry according to ASTM D4255 standard. The two rail shear test fixture has two rigid parallel steel rails for loading purpose. The rails are aligned to the loading direction as shown . Thus, it induces the shear load in the specimen which is bolted to these rails. A strain gage is bonded at 45° to the longitudinal axis of the specimen.

The Shear strength is obtained as

$$\tau_{xy}^{ult} = \frac{P_{\max}}{Lh}$$

where, $P_{\max}$ is ultimate failure load, L is the specimen length along the rails and h is the specimen thickness.

The shear modulus is given as

$$G_{xy} = \frac{\Delta\tau_{xy}}{\Delta\gamma_{xy}} = \frac{\Delta P}{2Lh\Delta\varepsilon_{45}}$$

where, ΔP is the change in applied load and $\Delta\varepsilon_{45}$ is the change in strain for $+45^\circ$ or -45° strain gage in the initial linear stress-strain regime. It is suggested that the change in the strain is taken as the average of the change in strains on the both sides of the specimen.

Various modes of failure are seen. The modes are highly dependent upon the microstructure of the material.

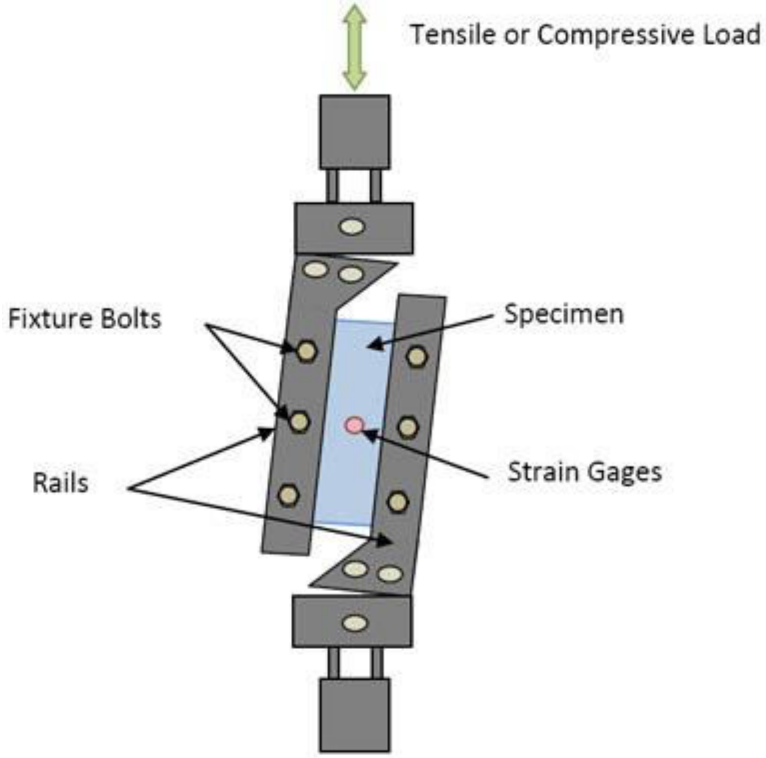


Figure 11: Two rail shear fixture for shear testing

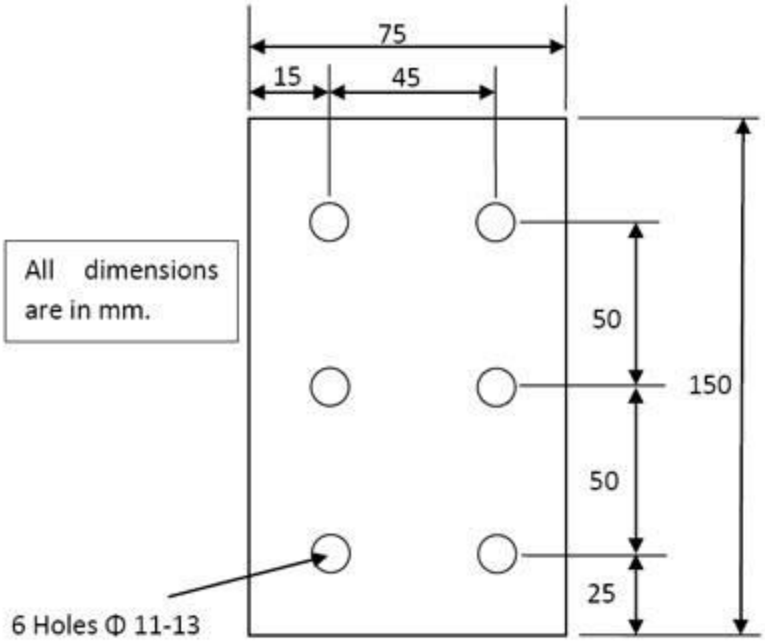


Figure 12: Two rail shear test specimen

***b. Three Rail (Symmetric) Shear Test*:**

The three rail shear test is an improved version of the rail shear test. Using one more rail in two rail shear test fixture it can produce a closer approximation to pure shear. The fixture consists of 3 pairs of rails clamped to the test specimen as shown . The outside pairs are attached to a base plate which rests on the test machine. Another pair (third middle) pair of rails is guided through a slot in the top of the base fixture. The middle pair loaded in compression. The shear force in laminate is generated via friction between rail and specimen. The strain gages bonded to the specimen at 45° to the specimen's longitudinal axis. The specimen geometry is shown.

The shear strength is given as

$$\tau_{xy}^{ult} = \frac{P_{\max}}{2Lh}$$

And the shear modulus is given as

$$G_{xy} = \frac{\Delta\tau_{xy}}{\Delta\gamma_{xy}} = \frac{\Delta P}{4Lh\Delta\varepsilon_{45}}$$

where, all variables in these two equations are given previously.

It should be noted that the holes in the specimen are slightly oversized than the bolts used for clamping. Further, the bolts are tightened in such a manner to ensure that there is no bearing contact between the bolt and specimen in the loading direction. It is recommended that each bolt is tightened with a 100 Nm torque.

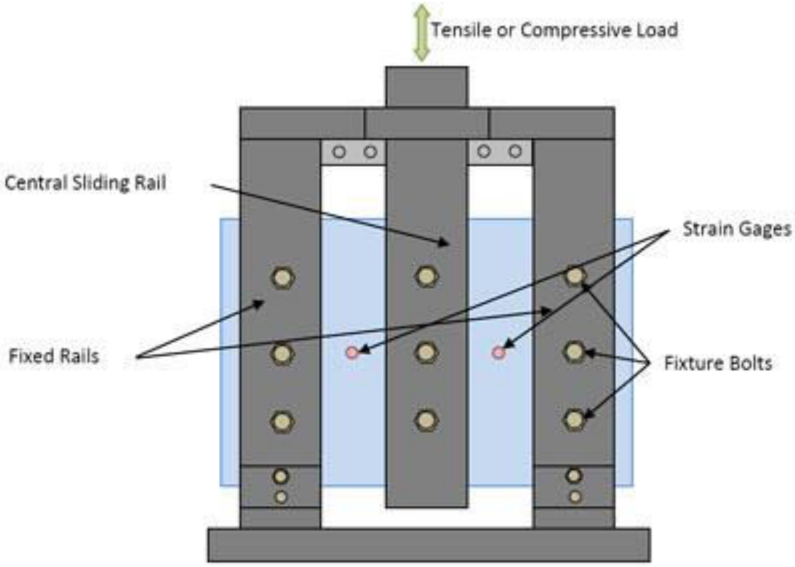


Figure 12: Three rail shear fixture for shear

testing

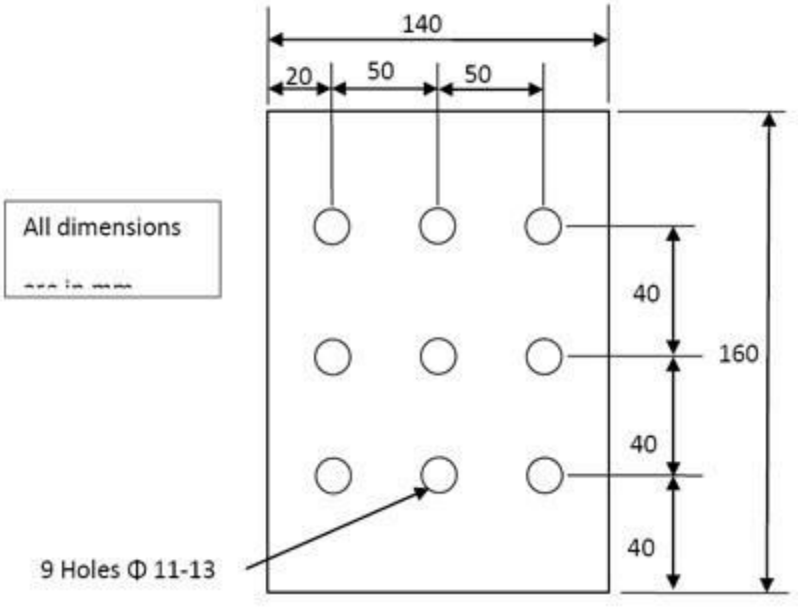


Figure 13: Specimen dimensions for three-rail shear test

Flexural Tests:

The flexural tests are conducted to determine the mechanical properties of resin and laminated fiber composite materials. Further, these tests are used to determine the interlaminar shear strength of a laminate, shear modulus, shear strength, tensile and compression moduli along with flexural and shear stiffness. These tests are not only used for composites but also for sandwich beams.

These tests are simple one. Further, they need simple instrumentation and equipment required. These tests conducted on beams of uniform cross section. These beam specimens do not require the end tabs.

There are two methods to carry out these tests. The beam is a flat rectangular specimen and is simply supported close to its ends. In the first method the beam is centrally loaded. Thus gives three point bending. Since there are three important points (two end supports and one central loading point) along the span of the beam this method is called as *three-point bending* test. In the second method the beam is loaded by two loads placed symmetrically between the supports. In this method there are four important points (two end supports and two loading points) along the span of the beam. Thus, it gives four-point bending. Hence, this method is called *four point bending*. These methods are shown schematically. Also shown in this figure are the shear force diagram (SFD) and bending moment diagrams (BMD) related to the particular loading regimes.

From the shear force and bending moment diagrams it is clear that there is a stress concentration at the point of loading. However, for four point bending there is uniform bending moment and both shear force and interlaminar shear stress are zero between the loading points. Thus, it leads to the pure bending loading. Such a state of stress is desirable in testing.

The properties are assumed to be uniform through the thickness as composite as it is a unidirectional composite or isotropic material. For such a material the normal stress varies linearly across the thickness. The maximum in compression is on one side and an equal maximum in tension on other side of the thickness and passes through zero at the mid-plane. The maximum normal stress is given as

$$|\sigma_C| = |\sigma_T| = \frac{6M}{bh^2}$$

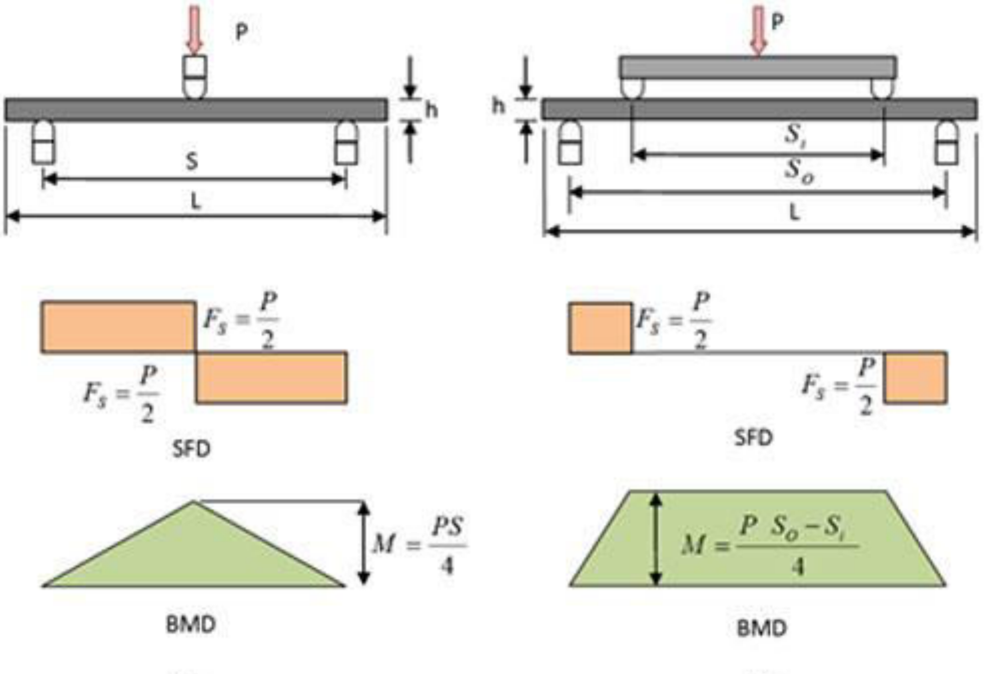
where, M is the bending moment, b is width and h is the thickness of the specimen. Further, σ_C and σ_T denote compressive and tensile normal stresses, respectively.

The shear stress varies parabolic through the thickness with maximum at mid plane and zero at the outer surface. The maximum shear stress at the mid plane is given as

$$\tau = \frac{3Fs}{2bh}$$

where Fs is the shear force on the specimen cross section. The normal stress and shear force variation through the thickness is shown.

The flexural response of the beam in three or four point bending test is obtained by recording the load applied and the resulting strain. The resulting strains are measured using the strain gages bonded on the beam in the gage length. It is clear from the distribution of the shear force and bending moment that the state of stress in specimens subjected to three and four-point bending tests are somewhat different. Thus, it may lead to differences in the results.



(a) (b)

Figure 13: Shear force and bending moment diagrams for (a) three point and (b) four point bending test

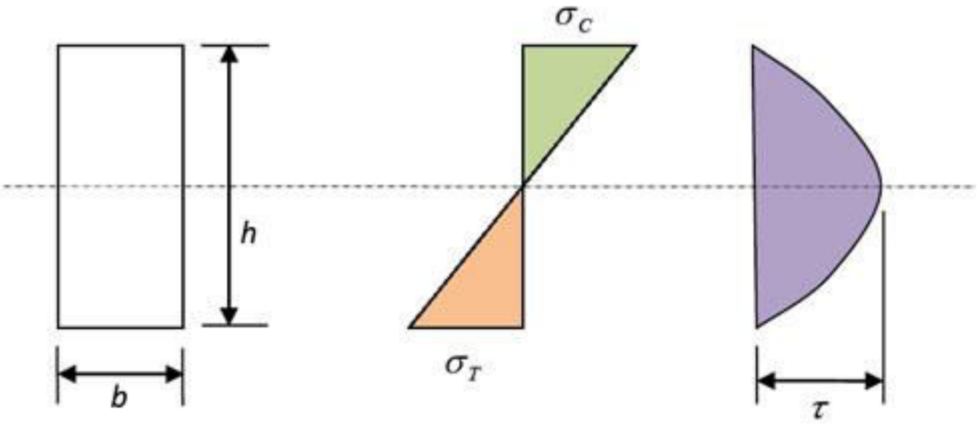


Figure 14: Bending and shearing stresses in the thickness direction

In the following we will see the measurement of flexural modulus and maximum stress on the outer surface of the beam.

Flexural strength**:** This is the stress on the surface of the specimen at failure, which should be accompanied by the breaking of fibers, rather than inter laminar shear.

In the three point bending method the flexural modulus E_f is given as

$$E_f = \frac{S^3 m}{4b\,h^3}$$

where, E_f is flexural modulus, S is the support span, m is the slope of the load-deflection curve, b and h are the width and thickness of the specimen, respectively.

In case of four point bending there are two options according to ASTM D790 standard. In the first option the loading span is one third of the support span. For this case the flexural modulus is given as

$$F_f = 0.21\frac{S^3 m}{b h^3}$$

In the second option the loading span is half of the support span. The flexural modulus for this case is given as

$$F_f = 0.17\frac{S^3 m}{b h^3}$$

where, the parameters in these two equations are as defined earlier.

The maximum stress on outer surface of the beam is given below for all the cases.

$$\sigma = \frac{3PS}{2b\,h^2} \quad \text{3 point bending}$$

$$\sigma = \frac{PS}{b\,h^2} \quad \text{4 point bending with loading span equal to one third support span}$$

$$\sigma = \frac{3PS}{4b\,h^2} \quad \text{4 point bending with loading span equal to one third support span}$$

It is important to note that the measurement of width and thickness of the beam is important for accurate measurement of flexural modulus and maximum stresses.

For more details on these tests one can refer to ASTM D790-92 and ASTM D790M-93.

UNIT-IV

LAMINATES

Plate stiffness and compliance

Assumptions

1. The laminate consists of perfectly bonded layers. There is no slip between the adjacent layers. In other words, it is equivalent to saying that the displacement components are continuous through the thickness.
2. Each lamina is considered to be a homogeneous layer such that its effective properties are known.
3. Each lamina is in a state of plane stress.
4. The individual lamina can be isotropic, orthotropic or transversely isotropic.
5. The laminate deforms according to the Kirchhoff - Love assumptions for bending and stretching of thin plates (as assumed in classical plate theory). The assumptions are:
    a. The normals to the mid-plane remain straight and normal to the midplane even after deformation.
    b. The normals to the mid-plane do not change their lengths.
6. The classical laminate theory is abbreviated as CLT. This theory is known as the classical laminated plate theory and abbreviated as CLPT.

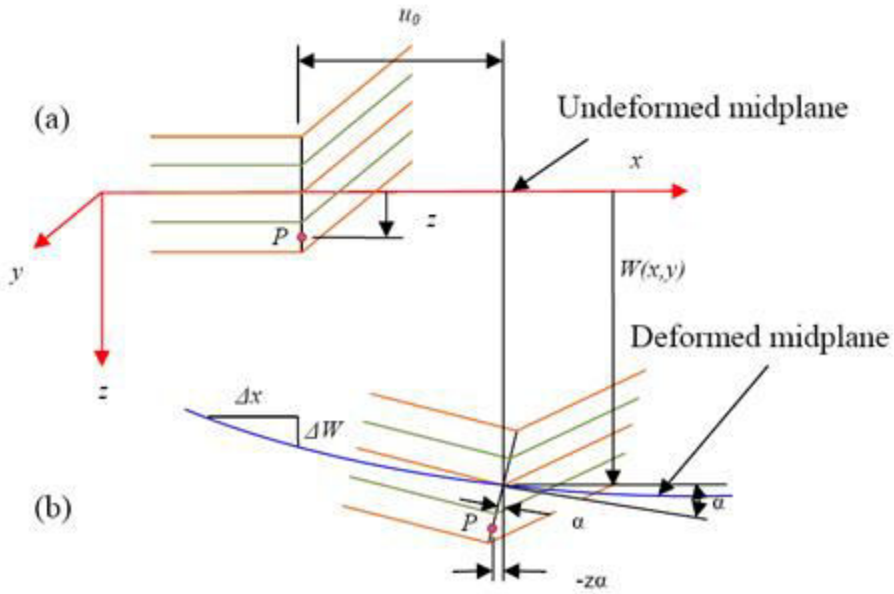


Fig: Plate deformation

$$u(x,y,z) = u_0(x,y) - z\tan\alpha = u_0(x,y) - z\,\alpha = u_0(x,y) - z\,\frac{\partial w}{\partial x}$$

Similarly, for the deformation in *yz* plane we can express the slope of the deformed mid-plane as. Thus, the displacement of a generic point along *y* axis can be given as

$$v(x,y,z) = v_0(x,y) - z\,\frac{\partial w}{\partial y}$$

Thus, the complete displacement field for a generic point in the laminate according to the classical laminate theory is given below:

$$u(x,y,z) = u_0(x,y) - z\,\frac{\partial w}{\partial x}$$
$$v(x,y,z) = v_0(x,y) - z\,\frac{\partial w}{\partial y}$$
$$w(x,y,z) = w_0(x,y)$$

From the first assumption of the Kirchhoff-Love theory that the normals remain straight and normal to mid-plane even after deformation, results into zero transverse shear strains. Thus,

$$\gamma_{xz} = \gamma_{yz} = 0$$

Using the definitions of small strain, we can write the above equation as

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0$$

From the first of the above equation we can write

$$\frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x}$$

Integrating this with respect to *z*, we get

$$u(x,y,z) = -z\frac{\partial w}{\partial x} + u_0(x,y)$$

Where $u_0(x,y)$ is a constant of integration which is function of *x* and *y* alone. Similarly, from the second of Equation (5.9), we can get

$$v(x,y,z) = -z\frac{\partial w}{\partial y} + v_0(x,y)$$

Strain Displacements Relations:

The strain displacement relations for infinitesimal strains using the displacement field can be given as

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$
$$\epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

The above equation can be written as

$$\begin{Bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\epsilon_{xx}^{(0)}\\ \epsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z\begin{Bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{Bmatrix}$$

or

$$\{\epsilon\}_{xy} = \{\varepsilon^{(0)}\}_{xy} + z\{\kappa\}_{xy}$$

$$\{\epsilon^{(0)}\}_{xy} = \{\epsilon_{xx}^{(0)} \quad \epsilon_{yy}^{(0)} \quad \gamma_{xy}^{(0)}\}^T = \left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T$$

$$\{\kappa\}_{xy} = \{\kappa_{xx} \quad \kappa_{yy} \quad \kappa_{xy}\}^T = \left\{-\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 w}{\partial y^2} \quad -2\frac{\partial^2 w}{\partial x \partial y}\right\}^T$$

The terms κ_{xx} and κ_{yy} are the bending moment curvatures and κ_{xy} is the twisting moment curvature.

Strain Displacements Relations:

The strain displacement relations for infinitesimal strains using the displacement field can be given as

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}$$

The above equation can be written as

$$\begin{Bmatrix}\epsilon_{xx}\\ \epsilon_{yy}\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\epsilon_{xx}^{(0)}\\ \epsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} + z\begin{Bmatrix}\kappa_{xx}\\ \kappa_{yy}\\ \kappa_{xy}\end{Bmatrix}$$

or

$$\{\epsilon\}_{xy} = \{\varepsilon^{(0)}\}_{xy} + z\{\kappa\}_{xy}$$

where $\{\epsilon^{(0)}\}_{xy} = \{\epsilon_{xx}^{(0)} \quad \epsilon_{yy}^{(0)} \quad \gamma_{xy}^{(0)}\}^T = \left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T$ are the midplane strains and $\{\kappa\}_{xy} = \{\kappa_{xx} \quad \kappa_{yy} \quad \kappa_{xy}\}^T = \left\{-\frac{\partial^2 w}{\partial x^2} \quad -\frac{\partial^2 w}{\partial y^2} \quad -2\frac{\partial^2 w}{\partial x \partial y}\right\}^T$ represents the midplane curvatures. The terms κ_{xx} and κ_{yy} are the bending moment curvatures and κ_{xy} is the twisting moment curvature.

State of Stress in a Laminate:

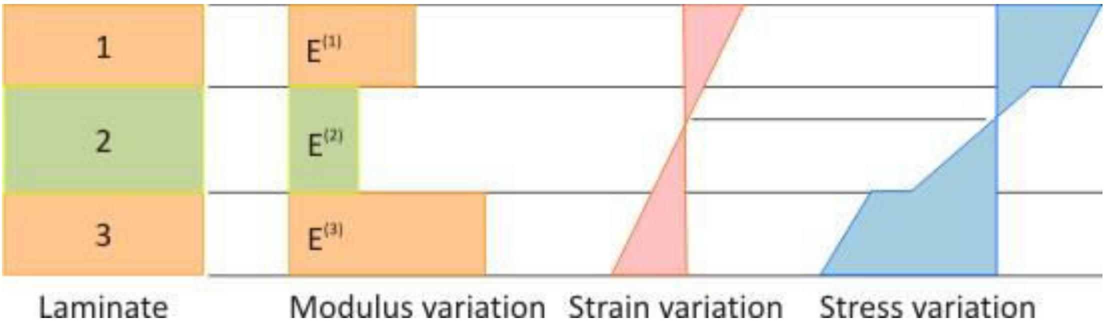
The stresses at any location can be calculated from the strains and lamina constitutive relations. It is assumed that the lamina properties are known. Hence, the constitutive equation for a *kth* lamina is known, that is, the reduced stiffness matrices (in principal material directions and global directions) are known. Thus, the stresses in k^{th} lamina can be given as

$$\{\sigma\}^k_{xy} = \left[\overline{Q}\right]^k \{\epsilon\}^k_{xy}$$

Now, using Equation (5.13), we can write the stresses as

$$\{\sigma\}^k_{xy} = \left[\overline{Q}\right]^k \left\{\epsilon^{(0)}\right\}_{xy} + \left[\overline{Q}\right]^k z\,\{\kappa\}_{xy}$$

. In these equations, the strains are given at a *z* location where the stresses are required. It should be noted that the strains are continuous and vary linearly through the thickness. If we look at the stress distribution through the thickness it is clear that the stresses are not continuous through the thickness, because the stiffness is different for different laminae in thickness direction. In a lamina the stress varies linearly. The slope of this variation in a lamina depends upon its moduli. However, at the interface of two adjacent laminae there is a discontinuity in the stresses. The same thing is depicted in below Figure with three layers.



Elucidation of stress discontinuity at lamina interfaces in a laminate

Inplane Resultant Forces:

The inplane forces per unit length are defined as

$$N_{xx} = \int_{-H}^{H} \sigma_{xx}\,dz, \qquad N_{yy} = \int_{-H}^{H} \sigma_{yy}\,dz, \qquad N_{xy} = \int_{-H}^{H} \tau_{xy}\,dz \qquad (5.16)$$

Or these can be written as

$$\{N\}_{xy} = \int_{-H}^{H} \{\sigma\}_{xy}\,dz$$

$$\{N\}_{xy} = \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \left\{\epsilon^{(0)}\right\}_{xy} dz + \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \{\kappa\}_{xy}\, z\, dz$$

Now recall that the midplane strains $\left\{\epsilon^{(0)}\right\}_{xy}$ and the curvatures $\{\kappa\}_{xy}$ are independent of *z* location. The reduced transformed stiffness matrix $\left[\overline{Q}\right]$ is function of thickness and constant over a given lamina thickness. Now we can replace the integration over the laminate thickness as sum of the integrations over individual lamina thicknesses. Thus, Equation can be written as

$$\{N\}_{xy} = \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \left\{\epsilon^{(0)}\right\}_{xy} dz + \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^k \{\kappa\}_{xy}\, z\, dz$$

Here, N_{Lay} is the total number of layers in the laminate. This equation can be written as

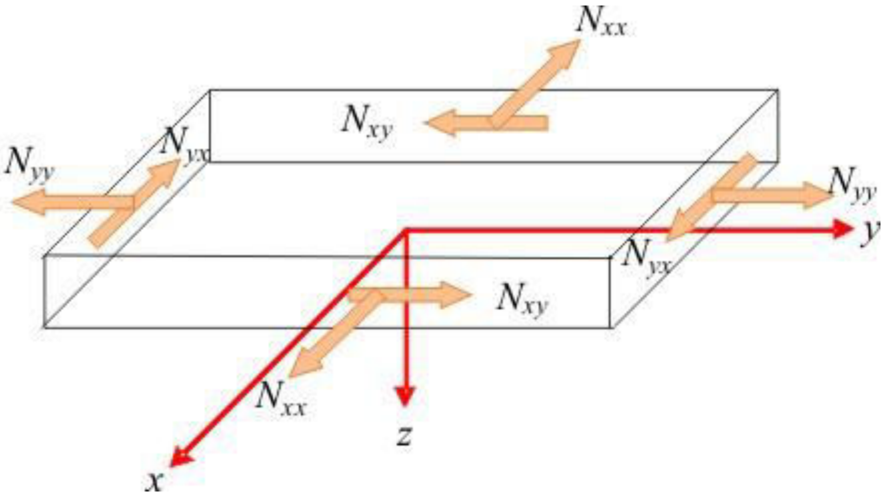
$$\{N\}_{xy} = [A]\left\{\epsilon^{(0)}\right\}_{xy} + [B]\{\kappa\}_{xy}$$

where

$$[A] = \sum_{k=1}^{N_{Lay}} \left[\overline{Q}\right]^k (z_k - z_{k-1}) \quad \text{and} \quad [B] = \frac{1}{2}\sum_{k=1}^{N_{Lay}} \left[\overline{Q}\right]^k (z_k^2 - z_{k-1}^2)$$

The matrix $[A]$ represents the in-plane stiffness, that is, it relates the in-plane forces with mid-plane strains and the matrix $[B]$ represents the bending stiffness coupling, that is, it relates the in-plane forces with mid-plane curvatures.

It should be noted that the matrices $[A]$ and $[B]$ are symmetric as the matrix $\left[\overline{Q}\right]$ is also symmetric for each lamina in the laminate.
The resultant in-plane forces are shown.



In plane resultant forces per unit length on a laminate

Resultant Moments:

The resultant moments per unit length are defined as

$$M_{xx} = \int_{-H}^{H} \sigma_{xx}\, z\, dz, \qquad M_{yy} = \int_{-H}^{H} \sigma_{yy}\, z\, dz, \qquad M_{xy} = \int_{-H}^{H} \tau_{xy}\, z\, dz$$

Or these can be written as

$$\{M\}_{xy} = \int_{-H}^{H} \{\sigma\}_{xy}\, z\, dz$$

Now, using Equation (5.15) we can write,

$$\{M\}_{xy} = \int_{-H}^{H} \left[\overline{Q}\right]^{k} \left\{\epsilon^{(0)}\right\}_{xy} z\, dz + \int_{-H}^{H} \left[\overline{Q}\right]^{k} \{\kappa\}_{xy}\, z^{2}\, dz$$

Now, with the same justification as given, we can write the above equation as

$$\{M\}_{xy} = \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^{k} \left\{\epsilon^{(0)}\right\}_{xy} z\, dz + \sum_{k=1}^{N_{Lay}} \int_{-z_{k-1}}^{z_k} \left[\overline{Q}\right]^{k} \{\kappa\}_{xy}\, z^{2}\, dz$$

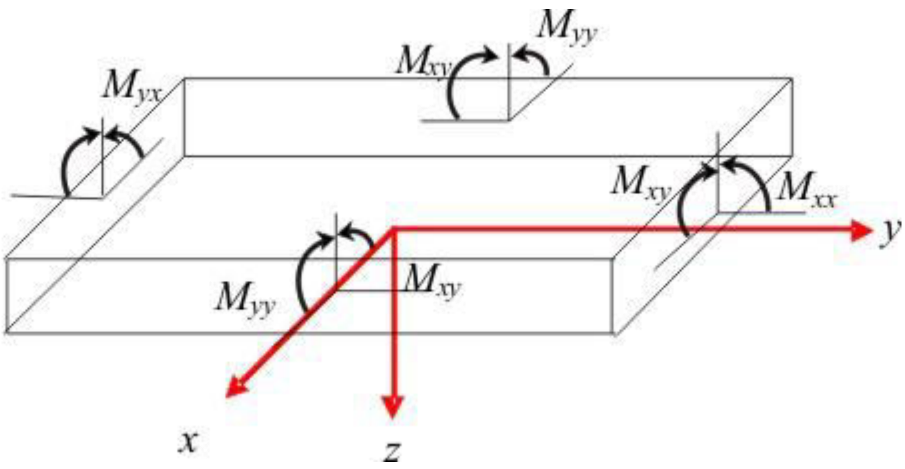
This can be written as

$$\{M\}_{xy} = [B]\left\{\epsilon^{(0)}\right\}_{xy} + [D]\{\kappa\}_{xy} \tag{5.26}$$

where

$$[D] = \frac{1}{3} \sum_{k=1}^{N_{Lay}} \left[\overline{Q}\right]^{k} \left(z_k^3 - z_{k-1}^3\right) \tag{5.27}$$

The matrix $[D]$ represents the bending stiffness, that is, it relates resultant moments with mid-plane curvatures. Again, the matrix $[D]$ is also symmetric. Further, it is important to note that the matrix $[B]$relates the resultant moments with mid-plane curvatures as well.



Resultant moments per unit length on a laminate

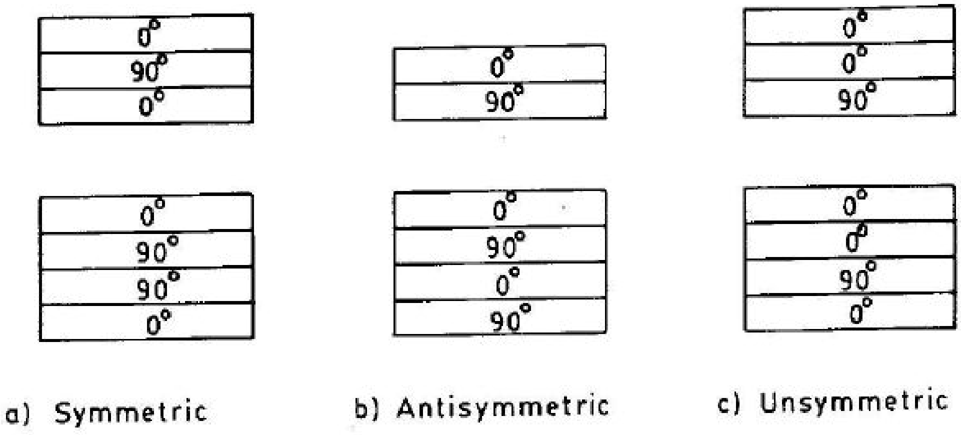
Types of Laminates:-

1. Based on Layer angle orientation

- Crossply Laminate
- Angle-ply Laminate

2. Based on layer orientation about midplane

- Symmetric Laminate
- Anti symmetric Laminate
- Un symmetric Laminate



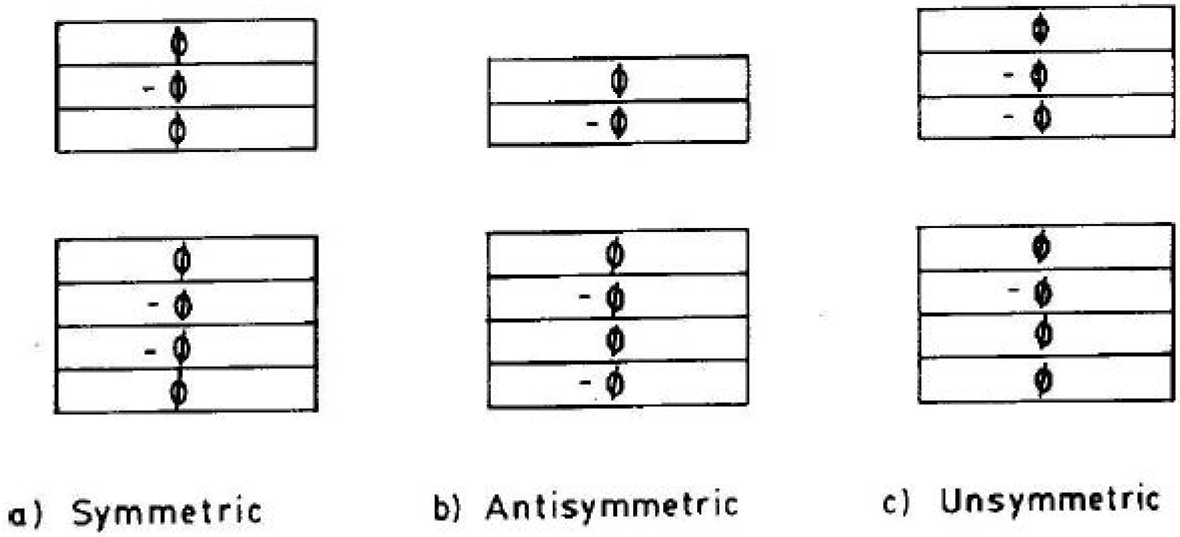
Cross-ply laminates

A laminate is called cross-ply laminate if all the plies used to fabricate the laminate are only 0° and 90°

For a cross ply laminate the terms $A_{16} = A_{26} = B_{16} = B_{26} = D_{16} = D_{26} = 0$. This is because these terms involve the terms $\overline{Q}_{16}$ and $\overline{Q}_{26}$ which have the products of *mn* terms. This product is zero for any cross-ply. Thus, the terms $\overline{Q}_{16}$ and $\overline{Q}_{26}$ are identically zero for each ply.

***Note*:** For a cross-ply following relations hold true. The readers should verify these relations from earlier lectures on planar constitutive relations.

$$\overline{Q}_{11}(0) = \overline{Q}_{22}(90), \quad \overline{Q}_{22}(0) = \overline{Q}_{11}(90)$$
$$\overline{Q}_{12}(0) = \overline{Q}_{12}(90), \quad \overline{Q}_{66}(0) = \overline{Q}_{66}(90)$$



a) Symmetric b) Antisymmetric c) Unsymmetric

Angle ply laminates

***Angle-Ply Laminates*:**
A laminate is called angle-ply laminate if it has plies of the same thickness and material and are oriented at $+\theta$ and $-\theta$.

For example $[45/-45/-30/30]$ is shown. For angle-ply laminates the terms $A_{16} = A_{26}$ are zero. This can be justified by that fact that $\overline{Q}_{16}$ and $\overline{Q}_{26}$ have the term *mn*. Due to this term $\overline{Q}_{16}$ and $\overline{Q}_{26}$ have opposite signs for layers with $+\theta$ and $-\theta$ fibre orientation. Since the thicknesses and materials of these layers are same, by the definition the terms $A_{16} = A_{26}$ are zero for the laminate.

***Note*:** For angle-ply laminates the following relations are very useful in computing $[A], [B]$ and $[D]$.

$$\overline{Q}_{11}(+\theta) = \overline{Q}_{11}(-\theta), \quad \overline{Q}_{22}(+\theta) = \overline{Q}_{22}(-\theta)$$
$$\overline{Q}_{12}(+\theta) = \overline{Q}_{12}(-\theta), \quad \overline{Q}_{66}(+\theta) = \overline{Q}_{66}(-\theta)$$
$$\overline{Q}_{16}(+\theta) = -\overline{Q}_{16}(-\theta), \quad \overline{Q}_{26}(+\theta) = -\overline{Q}_{26}(-\theta)$$

***Anti-symmetricLaminates*:**

A laminate is called anti-symmetric when the material and thickness of the plies are same above and below the mid-plane but the orientation of the plies at same distance above and below the mid-plane have opposite signs.

For example, $[45/-30/30/-45]$ is shown in Figure. For anti-symmetric laminates the terms $A_{16} = A_{26} = D_{16} = D_{26} = 0$. The proof is left to the readers as an exercise.

***BalancedLaminates*:**

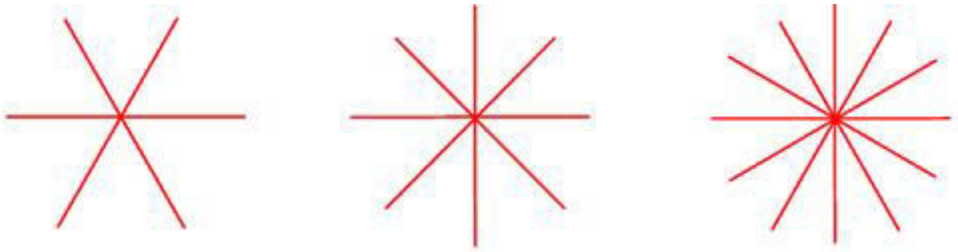
A laminate is called balanced laminate when it has pairs of plies with same thickness and material and the angles of plies are $+\theta$ and $-\theta$. However, the balanced laminate can also have layers oriented at 0° and 90°. For this laminate also $A_{16} = A_{26}$ are zero. It should be noted that angle-ply laminates are balanced laminates. For example, $[30/60/-45/-30/-60/45]$ is shown .

***Quasi-Isotropic Laminates*:**

A laminate is called quasi-isotropic when its extensional stiffness matrix behaves like an isotropic material. This requires that $A_{11} = A_{22}$, $A_{16} = A_{26} = 0$ and $A_{66} = (A_{11} - A_{12})/2$. Further, this extensional stiffness matrix is independent of orientation of layers in laminate. This requires a laminate with $N \geq 3$ equal thickness layers and *N* equal angles between adjacent fibre orientations. The *N* equal angles, $\Delta\theta$ between the fibre orientations in this case can be given as

$$\Delta\theta = \frac{\pi}{N}$$

The quasi-isotropic laminate with this construction for *N*=3, 4 and 6 will have fibre orientations as shown .



$N = 3\ (\Delta\theta = 60^\circ)$ $N = 4\ (\Delta\theta = 45^\circ)$ $N = 6\ (\Delta\theta = 30^\circ)$

Fibre orientations in a typical quasi-isotropic laminates

It should be noted that the isotropy in these laminates is in-plane only. The matrices *B* and *D* may not behave like an isotropic material. Hence, such laminates are quasi-isotropic in nature.

Some examples of quasi-isotropic laminate are: $[0/\pm 60]_S$, $[0/\pm 45/90]_S$

Example 1 Consider Example 5.3. Let this laminate be subjected to the forces $N_{xx} = 1000\ N/mm$, $N_{yy} = 500\ N/mm$ and $N_{xy} = 100\ N/mm$.

Calculate global strains and stresses in each ply.

***Solution*:** The laminate in this example is a symmetric laminate. Hence, *B* matrix is zero. It means that there is no coupling between extension and bending actions. Thus, the applied stresses will produce only in-plane and shear strains and it will not produce any curvatures. Thus, it is easy to understand that the mid-plane strains will be the strains in each ply.

We can find the mid-plane strains as follows:

$$\begin{aligned}\{N\} &= [A]\{\epsilon^{(0)}\} + [B]\{\kappa\}\\ &= [A]\{\epsilon^{(0)}\}\end{aligned}$$

This gives

$$\{\epsilon^{(0)}\} = [A]^{-1}\{N\}$$

Thus,

$$\begin{Bmatrix}\epsilon_{xx}^{(0)}\\ \epsilon_{yy}^{(0)}\\ \gamma_{xy}^{(0)}\end{Bmatrix} = 10^{-3}\begin{bmatrix}0.01120 & -0.00773 & 0\\ -0.00773 & 0.01120 & 0\\ 0 & 0 & 0.00759\end{bmatrix}\begin{Bmatrix}1000\\ 500\\ 100\end{Bmatrix} = 10^{-3}\begin{Bmatrix}7.335\\ -2.130\\ 0.759\end{Bmatrix}$$

The strains are same in all layers. However, the stresses in each layer will be different as their stiffnesses are different.

Stresses in $+45°$ layer are

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}_{(+45)} = 10^{-3}\begin{bmatrix}42.63 & 29.43 & 28.94\\ 29.43 & 42.63 & 28.94\\ 28.94 & 28.94 & 32.93\end{bmatrix}\begin{Bmatrix}7.335\\ -2.130\\ 0.759\end{Bmatrix} = \begin{Bmatrix}0.2719\\ 0.1471\\ 0.1756\end{Bmatrix} GPa$$

And stresses in $-45°$ layer are

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}_{(-45)} = 10^{-3}\begin{bmatrix}42.63 & 29.43 & -28.94\\ 29.43 & 42.63 & -28.94\\ -28.94 & -28.94 & 32.93\end{bmatrix}\begin{Bmatrix}7.335\\ -2.130\\ 0.759\end{Bmatrix} = \begin{Bmatrix}0.2281\\ 0.1031\\ -0.1256\end{Bmatrix} GPa$$

Now, let us find the strains and stresses in principal material directions as well for these laminae.

Let us transform the strains in $+45°$ layer as

$$\{\epsilon\}_{12} = [T_2(+45)]\{\epsilon\}_{xy}$$

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \\ -1.0 & 1.0 & 0.0 \end{bmatrix} \begin{Bmatrix} \epsilon_{xx}^{(0)} \\ \epsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 0.00298 \\ 0.00222 \\ -0.00946 \end{Bmatrix}$$

Similarly, the strains in $-45°$ layer in principal directions are

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 0.00222 \\ 0.00298 \\ 0.00946 \end{Bmatrix}$$

Now, stresses in principal directions in $+45°$ layer are

$$\{\sigma\}_{12} = [T_1(+45)]\{\sigma\}_{xy}$$

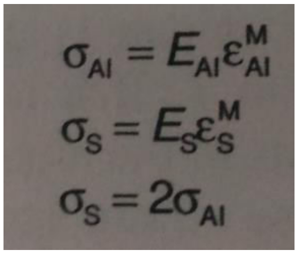
$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 0.29728 \\ 0.12166 \\ -0.12500 \end{Bmatrix} GPa$$

And stresses in principal material directions for $-45°$ layer are

$$\{\sigma\}_{12} = [T_1(-45)]\{\sigma\}_{xy}$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{Bmatrix} 0.29114 \\ 0.03990 \\ 0.06250 \end{Bmatrix} GPa$$

Hygrothermal Stresses



$$\sigma_{Al} = E_{Al}\varepsilon_{Al}^{M}$$

$$\sigma_{S} = E_{S}\varepsilon_{S}^{M}$$

$$\sigma_{S} = 2\sigma_{Al}$$

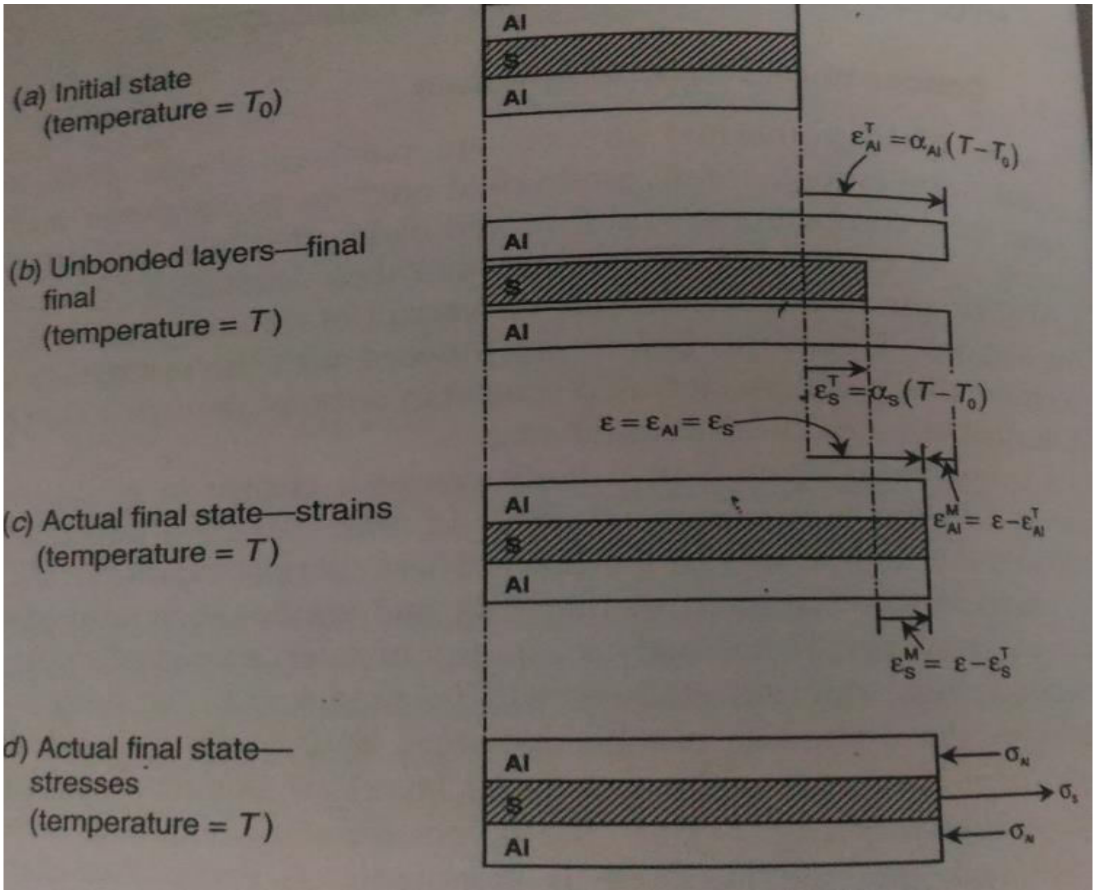


Fig: Thermal strain and stresses in a three-layered symmetric laminate

The strain due to change in temperature and moisture can be represented as

$$\varepsilon^{\mathrm{T}} = \alpha\,\Delta T$$

$$\varepsilon^{\mathrm{H}} = \beta\,\Delta C$$

Hygrothermal changes in longitudinal and transverse directions are

$$\varepsilon^{\mathrm{T}}_{\mathrm{L}} = \alpha_{\mathrm{L}}\,\Delta T$$

$$\varepsilon^{\mathrm{T}}_{\mathrm{T}} = \alpha_{\mathrm{T}}\,\Delta T$$

$$\varepsilon^{\mathrm{H}}_{\mathrm{L}} = \beta_{\mathrm{L}}\,\Delta C$$

$$\varepsilon^{\mathrm{H}}_{\mathrm{T}} = \beta_{\mathrm{T}}\,\Delta C$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = [\mathrm{T}_2]^{-1} \begin{Bmatrix} \alpha_{\mathrm{L}} \\ \alpha_{\mathrm{T}} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} = [\mathrm{T}_2]^{-1} \begin{Bmatrix} \beta_{\mathrm{L}} \\ \beta_{\mathrm{T}} \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^{\mathrm{T}} \\ \varepsilon_y^{\mathrm{T}} \\ \gamma_{xy}^{\mathrm{T}} \end{Bmatrix} = \begin{Bmatrix} \alpha_x \, \Delta T \\ \alpha_y \, \Delta T \\ \alpha_{xy} \, \Delta T \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^{\mathrm{H}} \\ \varepsilon_y^{\mathrm{H}} \\ \gamma_{xy}^{\mathrm{H}} \end{Bmatrix} = \begin{Bmatrix} \beta_x \, \Delta C \\ \beta_y \, \Delta C \\ \beta_{xy} \, \Delta C \end{Bmatrix}$$

The mechanical strains then are given as

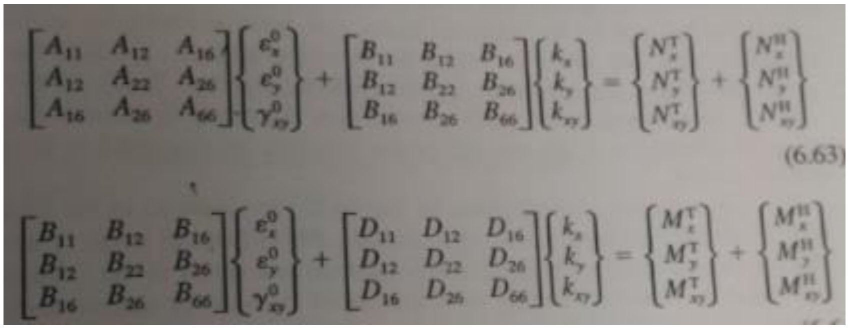
$$\begin{Bmatrix} \varepsilon_x^{\mathrm{M}} \\ \varepsilon_y^{\mathrm{M}} \\ \gamma_{xy}^{\mathrm{M}} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^{\mathrm{T}} \\ \varepsilon_y^{\mathrm{T}} \\ \gamma_{xy}^{\mathrm{T}} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^{\mathrm{H}} \\ \varepsilon_y^{\mathrm{H}} \\ \gamma_{xy}^{\mathrm{H}} \end{Bmatrix}$$

Using the above equations the mechanical strain can be rewritten as

$$\begin{Bmatrix} \varepsilon_x^{\mathrm{M}} \\ \varepsilon_y^{\mathrm{M}} \\ \gamma_{xy}^{\mathrm{M}} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 + zk_x \\ \varepsilon_y^0 + zk_y \\ \gamma_{xy}^0 + zk_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \, \Delta T \\ \alpha_y \, \Delta T \\ \alpha_{xy} \, \Delta T \end{Bmatrix} - \begin{Bmatrix} \beta_x \, \Delta C \\ \beta_y \, \Delta C \\ \beta_{xy} \, \Delta C \end{Bmatrix}$$

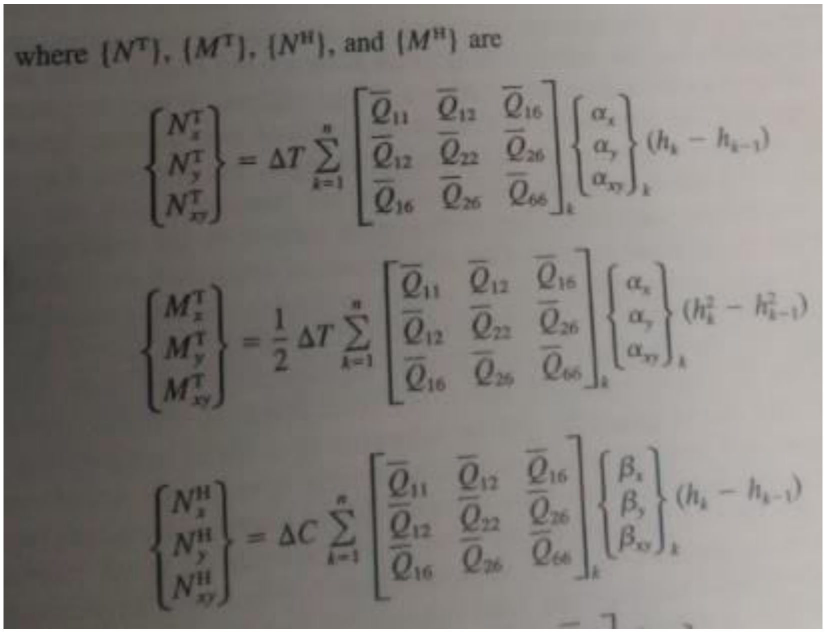
$$\begin{Bmatrix} \sigma_x^{\mathrm{T}} \\ \sigma_y^{\mathrm{T}} \\ \tau_{xy}^{\mathrm{T}} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 + zk_x - \alpha_x \, \Delta T - \beta_x \, \Delta C \\ \varepsilon_y^0 + zk_y - \alpha_y \, \Delta T - \beta_y \, \Delta C \\ \gamma_{xy}^0 + zk_{xy} - \alpha_{xy} \, \Delta T - \beta_{xy} \, \Delta C \end{Bmatrix}$$

Performing integration the resultant forces and moments are represented as below



$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} N_x^{\mathrm{T}} \\ N_y^{\mathrm{T}} \\ N_{xy}^{\mathrm{T}} \end{Bmatrix} + \begin{Bmatrix} N_x^{\mathrm{H}} \\ N_y^{\mathrm{H}} \\ N_{xy}^{\mathrm{H}} \end{Bmatrix} \tag{6.63}$$

$$\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} M_x^{\mathrm{T}} \\ M_y^{\mathrm{T}} \\ M_{xy}^{\mathrm{T}} \end{Bmatrix} + \begin{Bmatrix} M_x^{\mathrm{H}} \\ M_y^{\mathrm{H}} \\ M_{xy}^{\mathrm{H}} \end{Bmatrix}$$

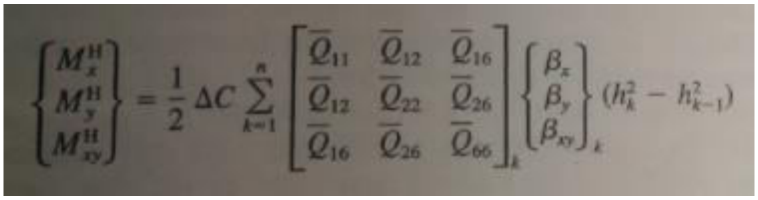


where $\{N^{\mathrm{T}}\}$, $\{M^{\mathrm{T}}\}$, $\{N^{\mathrm{H}}\}$, and $\{M^{\mathrm{H}}\}$ are

$$\begin{Bmatrix} N_x^{\mathrm{T}} \\ N_y^{\mathrm{T}} \\ N_{xy}^{\mathrm{T}} \end{Bmatrix} = \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (h_k - h_{k-1})$$

$$\begin{Bmatrix} M_x^{\mathrm{T}} \\ M_y^{\mathrm{T}} \\ M_{xy}^{\mathrm{T}} \end{Bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k (h_k^2 - h_{k-1}^2)$$

$$\begin{Bmatrix} N_x^{\mathrm{H}} \\ N_y^{\mathrm{H}} \\ N_{xy}^{\mathrm{H}} \end{Bmatrix} = \Delta C \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k (h_k - h_{k-1})$$



$$\begin{Bmatrix} M_x^{\mathrm{H}} \\ M_y^{\mathrm{H}} \\ M_{xy}^{\mathrm{H}} \end{Bmatrix} = \frac{1}{2} \Delta C \sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k (h_k^2 - h_{k-1}^2)$$

LAMINATE HYGROTHERMAL STRAINS

The changes in moisture concentration and temperature introduce expansional strains in each lamina. The stress-strain relation of an off-axis lamina is then modified as follows

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} + \begin{Bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \epsilon_6^e \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \epsilon_6^e \end{Bmatrix} = \begin{Bmatrix} \epsilon_1^H \\ \epsilon_2^H \\ \epsilon_6^H \end{Bmatrix} + \begin{Bmatrix} \epsilon_1^T \\ \epsilon_2^T \\ \epsilon_6^T \end{Bmatrix}$$

$$\begin{Bmatrix} \in_1^H \\ \in_2^H \\ \in_6^H \end{Bmatrix} = \Delta\overline{C} \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_6 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \in_1^T \\ \in_2^T \\ \in_6^T \end{Bmatrix} = \Delta T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_6 \end{Bmatrix}$$

where the superscripts e, H, T refer to expansion, moisture and temperature, respectively, ΔC and ΔT are the change in specific moisture concentration and temperature, respectively, and β's and α's are coefficients of moisture expansion and thermal expansion respectively.

Note that the spatial distributions of moisture concentration and temperature are determined from solution of moisture diffusion and heat transfer problems.
Expansional strains transform like mechanical strains i.e.,

$$\{\in'\} = [T_{\in}]\{\in\}$$

Inversion of Eq. 6.56 yields (see also Eq. 6.26), at any distance z ,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}_z = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_Z \begin{Bmatrix} \in_1 & - & \in_1^e \\ \in_2 & - & \in_2^e \\ \in_6 & - & \in_6^e \end{Bmatrix}_z$$

Thus, for a general laminate

$$\begin{Bmatrix} \mathrm{N}_1 \\ \mathrm{N}_2 \\ \mathrm{N}_6 \\ \mathrm{M}_1 \\ \mathrm{M}_2 \\ \mathrm{M}_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \in_1^0 \\ \in_2^0 \\ \in_6^0 \\ k_1 \\ k_2 \\ k_6 \end{Bmatrix} + \begin{Bmatrix} \mathrm{N}_1^e \\ \mathrm{N}_2^e \\ \mathrm{N}_6^e \\ \mathrm{M}_1^e \\ \mathrm{M}_2^e \\ \mathrm{M}_6^e \end{Bmatrix}$$

where the expansional force resultants are

$$\begin{Bmatrix} \mathrm{N}_1^e \\ \mathrm{N}_2^e \\ \mathrm{N}_6^e \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_z \begin{Bmatrix} \in_1^e \\ \in_2^e \\ \in_6^e \end{Bmatrix}_z dz$$

and the expansional moments are

$$\begin{Bmatrix} \mathrm{M}_1^e \\ \mathrm{M}_2^e \\ \mathrm{M}_6^e \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}_z \begin{Bmatrix} \in_1^e \\ \in_2^e \\ \in_6^e \end{Bmatrix}_z z\, dz$$

These expansional force resultants and moments may considerably influence the deformation behaviour of a laminate.

UNIT-V

JOINING METHODS AND FAILURE THEORIES

Stiffness of Aligned Short Fibre Composites

We can use the same methodology to obtain the elastic modulii of the short fibre composite as we did for the continuous fibre composite. Using Hookes' Law we can substitute E□ for the stress in each component. In the matrix, the strain is uniform and equal to that in the composite, in th eshort fibres, the strain varies with position along the fibre but averages out, along each fibre, to the strain in the matrix. Hence after canceling the strain terms we end up with.

- The stifness when $l < l_c$

$$E = \left(1 - f\left(1 - \frac{l}{2d}\right)\right)E_m$$

- The stifness when $l > l_c$

$$E = fE_f\left(1 - \frac{l_c}{2l}\right) + (1 - f)E_m$$

The stiffness measured perpendicular to the fibre axis is just the same as the continuous fibre case that we looked at earlier. But what about where the fibres are oriented in random directions ? Since random orientations occur quite commonly in composites which are made by spraying a combination of chopped fibres and resin onto a mould form we need a different method for estimating the elastic modulus.

Orientation Distributions

How do we calculate the elastic properties of a short fibre composite when the fibres are not aligned but have a distribution of orientations.

Strength of Short Fibre Composites

In short fibre composites - such as dough and sheet moulding compounds, or even chopped fibre reinforced mmc castings, the strength of the composite will depend on the length of the fibres and the orientation as well as the volume fraction. It is perhaps one of the great advanages of the injection moulding - squeeze casting fabrication route (possibly forging) that the dies can be made such that the flow of matrix is approximately parallel to the direction which will experience the greatest tensile stress. Given the fibre morphology, the bulk of the fibres will align in the flow direction and the effective volume fraction of fibres can range from 50 to almost 100% of the nominal fibre loading.

If the fibres are of optimum length then the matrix and fibre will fail simultaneously, the fibre experiencing its fracture stress at the midpoint of the fibre, with the average stress carried in the fibre being one half that value. Using the isostrain rule we find

Isostrain fibre stress at the matrix failure strain $= \varepsilon_m E_f$

$$\sigma_c = f\bar{\sigma}_f + (1 - f)\cdot\sigma_m(\varepsilon_m) = f\frac{\sigma_f}{2} + (1 - f)\cdot\sigma_m(\varepsilon_m)$$

Average stress in the fibre at the strain at which the matrix fails

Matrix failure stress

where f, in this case, is the actual fibre loading parallel to the tensile stress. If the fibres are longer than the critical length then the strength in the composite will depend on whether the matrix or fibres fail first and the arguments developed in the previous two sections can be followed if $\sigma_f(1 - l_c/(2l))$ is substituted for σ_f

$$\sigma_c = f\bar{\sigma}_f + (1-f)\cdot\sigma_m(\varepsilon_m) = f\left(1 - \frac{l_c}{2l}\right)\sigma_f + (1-f)\cdot\sigma_m(\varepsilon_m)$$

Isostrain fibre stress at matrix failure strain

$$\sigma_c = f\left(1 - \frac{l_c}{2l}\right)E_f\,\varepsilon_m + (1-f)\cdot\sigma_m(\varepsilon_m)$$

Failue stress of matrix

Assumes Linear Elastic Fibre

- How do we calculate the strength of a short fibre composite when the fibres are not aligned but have a distribution of orientations.

12 Halpin-Tsai Equations

The Halpin-Tsai equations are a set of empirical relationships that enable the property of a composite material to be expressed in terms of the properties of the matrix and reinforcing phases together with their proportions and geometry. These equations were curve fitted to exact elasticity solutions and confirmed by experimental measurements - they work well but the parameter ζ has no scientific basis nor is it related to any material or geometric property. Halpin and Tsai showed that the property of a composite P_c could be expressed in terms of the corresponding property of the matrix P_m and the reinforcing phase (or fibre) P_f using the following relationships:-

$$P_c = P_m\left(\frac{1+\zeta\eta f}{1-\eta f}\right) \qquad \eta = \frac{\left(\frac{P_f}{P_m}\right)-1}{\left(\frac{P_f}{P_m}\right)+\zeta}$$

The factor ζ is used to describe the influence of geometry of the reinforcing phase on a particular property. This factor is different for different properties in the same composite. The table below summarizes this factor for many typical geometry's.

Geometry	E_x	E_y	ν	G
Aligned continuous fibres	$fE_f + (1-f)E_m$	$\frac{E_f E_m}{fE_m + (1-f)E_f}$ or $\zeta = 2 + 40f^{10}$	$f\nu_f + (1-f)\nu_m$	$\zeta = 1 + 40f^{10}$ or $G_m\left(\frac{G_f(1+f) + G_m(1-f)}{G_m(1+f) + G_f(1-f)}\right)$
Spherical particles	$\zeta = 2 + 40f^{10}$	$\zeta = 2 + 40f^{10}$	$f\nu_f + (1-f)\nu_m$	$\zeta = 1 + 40f^{10}$

Oriented short fibres	$l < l_c \quad E_m\left(1-f\left(1-\frac{l}{2d}\right)\right)$ $l \geq l_c \quad fE_f\left(1-\frac{l_c}{2l}\right)+(1-f)E_m$	$\zeta = 2 + 40f^{10}$	$fv_f + (1-f)v_m$	$\zeta = 1 + 40f^{10}$
Oriented plates	$\zeta = 2\left(\frac{l}{t}\right) + 40f^{10}$	$\zeta = 2\left(\frac{w}{t}\right) + 40f^{10}$	$fv_f + (1-f)v_m$	$\zeta = \left(\frac{l+w}{2t}\right)^{1.73} + 40f^{10}$
Oriented whiskers	$\zeta = 2\left(\frac{l}{d}\right) + 40f^{10}$	$\zeta = 2 + 40f^{10}$	$fv_f + (1-f)v_m$	$\zeta = \left(\frac{l}{d}\right)^{1.73} + 40f^{10}$

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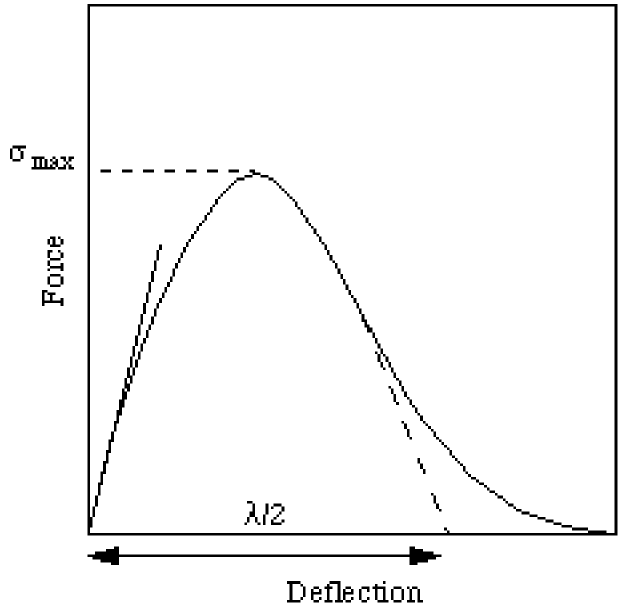
In all composite systems the equations are not valid above f=0.9 since these volume fractions of fibres are impossible geometrically. Linear Elastic Fracture Mechanics

It is possible to determine the ideal fracture strength of a material by equating the work done in separating unit area of atoms (i.e. breaking the interatomic (intermolecular) bonds) with the energy associated with the two new surfaces.



Since the work done is the product of force and distance, the work done seperating unit area is the product of force/unit area (stress) and distance. As stress (□) is a function of distance the work done/unit area is found by taking the integral of □(x) with respect to x (distance).

$$W = \int_0^{\lambda/2} \sigma(x)dx$$

Approximate the force - deflection curve to a sinewave such that

$$\sigma = \sigma_{\max} \sin(\frac{2\pi x}{\lambda})$$

Now, solving the integral we can determine the work of fracture and equate this to the surface energy (□).

$$\sigma_{\max} = \frac{2\pi\gamma}{\lambda}$$

In the elastic region we can assume a linear elastic material where stress and strain are related through Hooke’s Law

$$\sigma = E\varepsilon$$

$$\sigma_{\max} \sin(\frac{2\pi x}{\lambda}) = \frac{Ex}{b}$$

For small angles sin(x)=x and hence to a first approximation we find

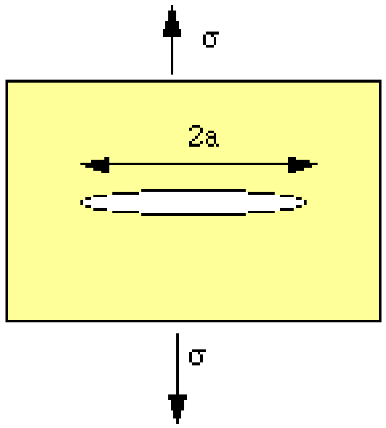
$$\sigma_{\max}\left(\frac{2\pi x}{\lambda}\right) = \frac{Ex}{b}$$

$$\sigma_{\max}\left(\frac{\sigma_{\max} x}{\gamma}\right) = \frac{Ex}{b}$$

$$\sigma_{\max} = \sqrt{\frac{E\gamma}{b}}$$

which for the majority of crystalline materials gives strengths of the order of E/10.

Unfortunately, it is vary rare that monolithic materials can acheive such strengths.



Griffith (A.Griffith Phil.Trans. Roy. Soc. A221, 163 (1920)) proposed that the much lower experimentally determined strengths of brittle solids such as ceramics, where there is little plasticity because of the difficulty of moving dislocations, were the result of the presence within the materials of a population of crack-like defects each of which was capable of concentrating the stress at its crack tip. The magnitude of the stress concentration was dependent on the crack length and the applied stress. Failure would occur when the stress local to the largest crack exceeded the theoretical fracture strength even though the macroscopic stress was relatively low.

In order to determine the magnitude of the Griffith effect we can consider a simple defect - an ellipitical crack of length 2a oriented perpendicular to the maximum principal stress. The concentrated value of the stress at each end of the ellipse would be

$$\sigma_{\max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

where □ is the radius of curvature of the crack tip - for an atomically sharp crack, the radius of curvature is similar in magnitude to the burgers vector of a dislocation, and the ratio a/□ is subsequently much greater than 1. So...

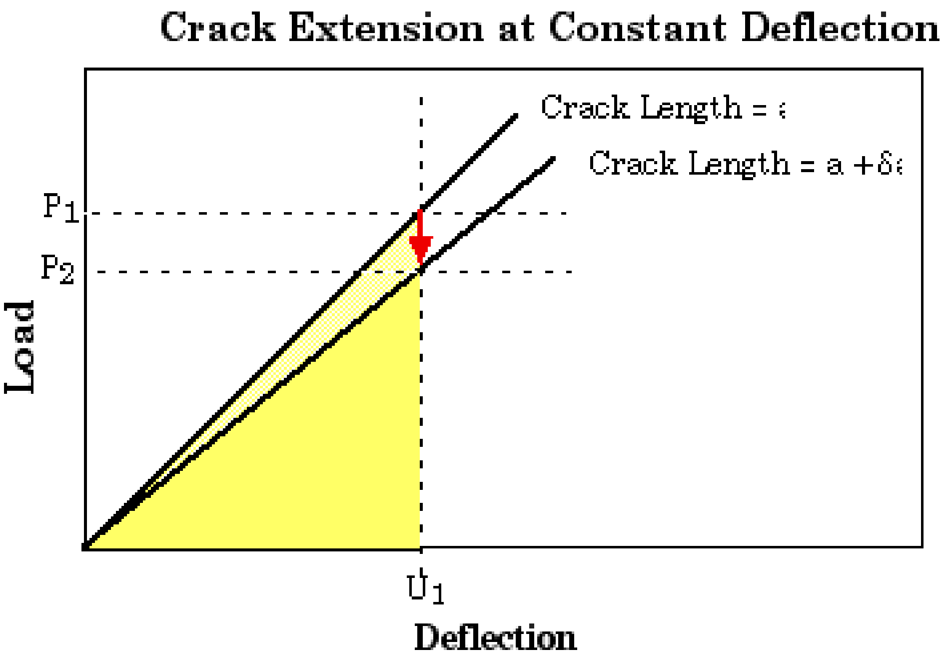
$$\sigma_{cracktip} = 2\sigma\sqrt{\frac{a}{b}} = \sqrt{\frac{E\gamma}{b}}$$
$$\sigma_{\max} = \sqrt{\frac{E\gamma}{4a}}$$

It should be seen immediately that a defect of about 1mm in length is sufficient to reduce the fracture stress by 2 orders of magnitude.

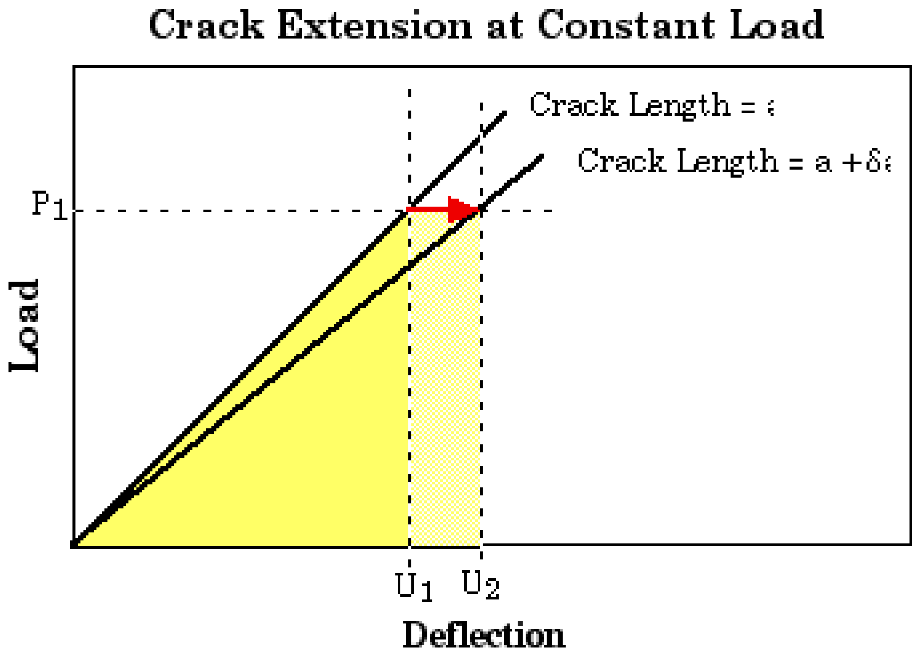
Griffith's main achievement in providing a basis for fracture strengths of materials containing cracks was his realisation that it was possible to derive a thermodynamic criterion for fracture by considering the change in energy of the material as a crack in it increased in length. Only if the total energy decreased would the crack extend spontaneously under the applied stress. The value of the energetic approach to fracture is that it defocuses attention from the microscopic details of deformation and fracture in the immediate vicinity of the crack tip.

Consider a crack of length 2a is situated in an infinite body and is oriented normal to the applied stress s. Now let us evaluate the changes in energy that occur as the crack is extended by a small distance, □a.

Firstly, new crack surfaces are created - absorbs energy - 2 surfaces of area w□a multiplied by □, the surface energy per unit area. Secondly, as the crack is assumed to advance only a small amount, the stress and displacement at the crack tip are unchanged. However, these are not the only source of changes in energy. We should consider the macroscopic load displacement curves for a material with a crack of length a and length (a+□a).



The material with the larger crack behaves like a weaker spring. Under conditions where a there is a fixed deflection, the extension of the crack is accompanied by a reduction in the load. Thus there is a reduction in the stored elastic strain energy in the body from $1/2P_1u_1$ to $1/2P_2u_1$ because at the same displacement the weaker spring requires less load. Thus at constant deflection the extension of the crack results in a decrease in the elastic strain energy of $1/2(P_1-P_2)u_1$ and increase in the surface energy of 2□w□a - where w is the thickness of the sample.



If, however, we now consider the conditions of constant load the situation is slightly more complicated but as we shall demonstrate the nett effect is the same. Here the weaker spring will extend more under a constant load there is thus an increase in the elastic strain energy from $1/2P_1u_1$ to $1/2P_1u_2$. However, since there is an extension in the sample, the applied load must fall from u_1 to u_2 and thus there is a decrease in the potential energy of the load from Pu_1 to Pu_2. Thus the energy in the material has decreased by an amout $P_1(u_2-u_1)-1/2P_1(u_2-u_1)$.

Thus under condition of constant load there is a reduction in potential energy while under conditions of constant deflection there is a reduction in stored elstic strain energy. Now, the strain energy released = -1/2udP and the potential energy released = -Pdu +1/2Pdu = -1/2pdu

The relationship between deflection u and load P is gibven by

$$u=CP$$

where C is the compliance of the system and

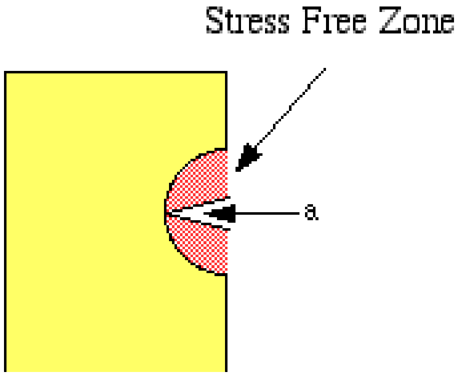
$$du=CdP$$

Now substitute for u in the strain energy released and for du in the potential energy released we see that the two are identical with the change in energy being

$$-1/2CPdu.$$

So where is all this leading?

Well, Griffith recognised that the driving force (thermodynamics again) for crack extension was the difference between the reduction in elastic strain energy/potential energy and that required to create the two new surfaces. Simple! Well almost.



The total energy change

W = - the strain energy(U) + the surface energy (S)

From the figure above, we see

U = -1/2 x stress x strain x stress free area x thickness

$$U = \frac{1}{2} \cdot \sigma \cdot \frac{\sigma}{E} \cdot \frac{\pi a^2}{2} \cdot t$$

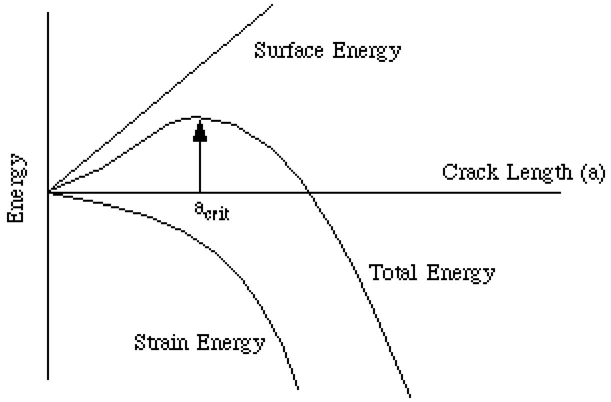
$$= \frac{\sigma^2 \pi a^2}{4E}$$

for unit thickness,while S is

$$S = 2\gamma a$$

for unit thickness

The crack will propogate when any impending increase in length results in a decrease in total energy, i.e. when any crack is longer than acrit, this is simply the value of a at the maximum in the total energy curve i.e. where dW/da=0,



$$\sigma = 2\sqrt{\frac{E\gamma}{\pi a}}$$

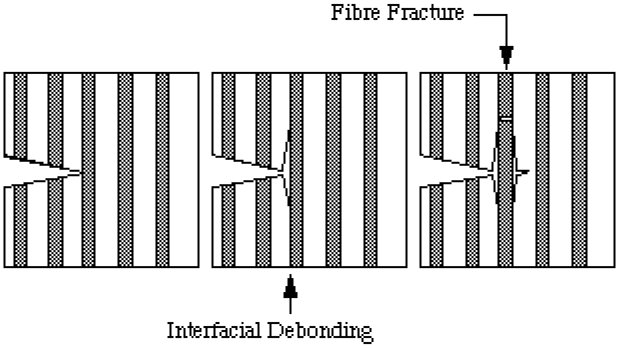
$$\sigma = 2\sqrt{\frac{E\gamma}{\pi\left(1 - \upsilon^2\right)a}}$$

In very brittle solids the term γ usally takes the value of the surface energy. However, where there are other energy absorbing processes taking place at the crack tip, such as dislocation motion (plasticity), then γ should be replaced with G, the strain energy release rate.

Continue with an exploration of the toughening effect of Fibre Pullout in short fibre composites.

14 Toughness in Composites - Part 2

Interfacial Fracture - Fibre Matrix Debonding



In a continuous fibre composite it is unlikely that all the fibres will have to be pulled out from the matrix since the fibres often fracture. Due to the statistical nature of the defect distribution in the surface of the fibres, not all fibres will wish to break in the plane of the crack. If the bonding between the fibre and matrix is weak then since the fibres are carrying the bulk of the stress at the crack tip, there will be a greater poisson's contraction in the fibre than in the matrix and as such a tensile stress will develop perpendicular to the interface between the fibre, which is contracting and the matrix which is not. This stress can fracture the weak fibre matrix interface and the crack is forced to run up, down and around the fibres. In order for the crack to proceed past the fibre, the fibre must break. This only occurs when the stress in the debonded fibre is raised to the fracture strength of that fibre - recall the statistical distribution of fibre strengths so this stress may be less the maximum value of sf. This requires an additional amount of elastic strain energy to be input into the debonded region of the fibre at the crack tip - energy which is released as heat and noise as the fibre fractures. Since the fibres are linearly elastic, the elastic strain energy per-unit volume of fibre is

$$G_\varepsilon = \frac{\sigma_f^2}{2E_f}$$

where $\square_f$ is the fracture strength of the fibre and E_f, the elastic modulus of the fibre. The total additional energy required is the product of the number of fibres per unit area, the additional strain energy per unit volume of fibre and the volume of debonded fibre, ie.

$$\begin{aligned} G_\varepsilon &= f\,\frac{4}{\pi d^2} \times \frac{\pi d^2}{4} \times l \times \frac{\sigma_f^2}{2E_f} \\ &= \frac{f\sigma_f^2 l}{2E_f} \end{aligned}$$

If we further assume that the crack running along the interface is limited to a length no shorter than the critical fibre length ie. $l_c = d\,\frac{\sigma_f}{\sigma_m}$ then

$$G_\varepsilon = \frac{fd\sigma_f^3}{2E\sigma_m}$$

to which we must add the additional surface area of the fibre-matrix debond mutliplied by the surface energy (x2 for the two new surfaces created).

$$G_s = f\frac{4}{\pi d^2}\times \pi dl \times 2\gamma = 8f\frac{l}{d}\gamma$$

$$G = f\left(\frac{d\sigma_f^3}{2E\sigma_m} + \frac{8\sigma_f\gamma}{\sigma_m}\right)$$

Example Problem

Estimate the contribution to toughening of interfacial fracture relative to fibre pullout in the carbon fibre epoxy composite studied in the previous example.

G = 0.6/2 x $8\text{x}10^{-6}$ x $(1800\text{x}10^{6})^{3}$ /(2 x $290\text{x}10^{9}$ x $85\text{x}10^{6}$) + (8 x (0.6/2) x$1800\text{x}10^{6}$ / $85\text{x}10^{6}$) = 335 Jm^{-2}.

15 Origin of Toughness in Composites

The most significant property improvement in fibre reinforced composites is that of fracture toughness. Toughness is quantified in terms of the energy absorbed per unit crack extension and thus any process which absorbs energy at the crack tip can give rise to an increase in toughness. In metallic matrices, plastic deformation requires considerable energy and so metals are intrinsically tough. In fibre reinforced materials with both brittle fibres and brittle matrices, toughness is derived from two sources. Firstly, if the crack can be made to run up and down every fibre in its path the there will be a large amount of new surface created for a very small increase in crack area perpendicular to the maximum principal stress - INTERFACIAL ENERGY - and in order to get the fibres to break they have to be loaded to their fracture strength and this often rquires additional local elastic work, and secondly If the fibres do not break and therefore bridge the gap then work must be done to pull the fibres out of the matrix - FIBRE PULLOUT. Using simple geometric models we can estimate the contribution of each of these processes to the overall toughness of the composite.

Fibre Pullout - Discontinuous Fibres

Consider the propagation of a crack through a matrix containing short fibres of length lc such that the fibres cannot break. The fibres will bridge the crack and for the crack to extend it is necessary to pull the fibres out of the matrix. Thus the stored elastic strain energy must do work pulling out the fibres against friction or by shearing the matrix parallel to the fibres as well as driving the crack through the matrix. We can estimate the work done pulling out a single fibre by integrating the product F(x).x (force x distance) over the distance $l_c/2$, where F(x) is the force - distance equation given by the shear lag model.

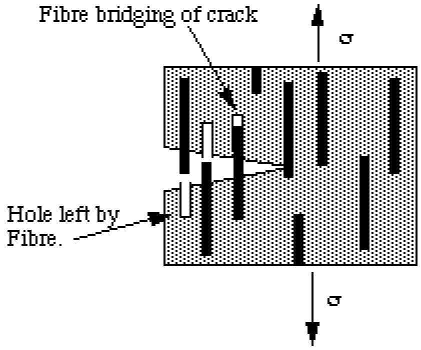


Figure 1. Fibre pullout during crack growth.

$$W = \int_0^{l/2} F(x)\,dx$$

$$? = \int_0^{l/2} \pi d \frac{\sigma_m}{2} x\,dx = \left[\frac{\pi d \sigma_m x^2}{4}\right]_0^{\frac{l}{2}}$$

$$? = \frac{\pi d \sigma_m l^2}{16}$$

In the above equation d is the fibre diameter, $\square_m$ the matrix yield strength and l, the fibre length. The number of fibres intersecting unit area of crack is simply dependent on the volume fraction,

$$N = \frac{4f}{\pi d^2}$$

hence the total work done, G, in extending the crack unit area is

$$G = \pi d \sigma_m \frac{l^2}{16} \times \frac{4f}{\pi d^2}$$

$$? = \frac{f \sigma_m l^2}{4d}$$

$$G_{max} = \frac{f \sigma_m l_c^2}{4d} = \frac{fd}{4}\left(\frac{\sigma_f^2}{\sigma_m}\right)$$

The longest fibre that can be pulled out is the critical fibre length, l□, which in turn depends on the fibre fracture strength, $\square_f$. Thus a combination of strong fibres in a relatively weak fibre/matrix interface give the best toughness.

Continuous Fibres

What happens when the fibres are continuous - will the Strain energy relaease rate saturate at its maximum value shown above ? In a composite designed such that the majority of the load is carried by the fibres, the stress in the fibres will increase uniformly with strain upto the fibre uts. Since the strains in the presence of a crack are non-uniform, being greatest at the crack tip then the stress in the fibre will be greatest at the crack tip and so the fibre will fracture in the plane of the crack - hence no pullout. If however the fibres contain a population of defects - as real fibres do - with an average strength of $\square_f$* and a spacing of l then it is possible for some of the continuous fibres to fracture within a distance l_c* /2 of the crack plane and be pulled out as the crack advances - note that we must reduce l_c to l_c* on account of the lower effective fibre strength, $\square_f$* . ie.

$$\boxed{l_c^* = \left(1 - \frac{\sigma_f^*}{\sigma_f}\right) l_c}$$

if the defect has no strength then l_c*=l_c and the fibre acts as a short fibre of length l. If the defect as a strength almost equal to that of the ideal fibre then l_c* =0 and no pullout is possible. The fraction of fibres pulling out will be l_c* /l (recall l is the average distance between defects) then the work done in pulling out the fibres is

$$\boxed{G = \frac{l_c^*}{l}\left(\frac{f\sigma_m}{4d}\right) l_c^{*2} = \left(1 - \frac{\sigma_f^*}{\sigma_f}\right)^3 \frac{f\sigma_m}{4ld} l_c^3}$$

We can also use this expression to determine the toughness of short fibre composites of length l since these corespond to continuous fibres with defect spacings of l in which the defects have zero strength . ie for short fibres of length l>l_c we have

$$\boxed{G = \frac{f\sigma_m}{4ld} l_c^3}$$

Example Problem

Estimate the maximum work of fracture in a carbon short-fibre epoxy composite containing 60 vol% carbon fibres and hence determine the fracture toughness K_{Ic} - assume only 50% of the fibres are parallel to the loading direction and have a length equal to the critical length..

Data:

Strength of Carbon fibre = 1800 MPa (Amoco Chemicals T650)
Strength of Epoxy = 85 MPa (Ciba Geigy Araldite HM94)
Fibre Diameter = 8μm
Modulus Data from previous example.

(i) Maximum Energy Absorbtion

$G = fd\square_f^2 / 4\square_m = (0.6/2) \times 8\times10^{-6} \times (1.8\times10^9)^2 / (4 \times 85\times10^6) = 23\ \mathrm{kJm}^{-2}$.

(ii) Fracture Toughness:- *First evaluate the modulus of the composite*

$E = f^*E_f (1-l_c/2l) + (1-f^*) E_m = 0.3x290 (1-1/2) + 2.8x0.7 = 45$ GPa

$K^2=GE = 23x10^3$ x $45x10^9 = 1.04x10^{15}$. *ie.* K = 32 MPa.m$^{1/2}$

The value obtained for the fracture toughness of the composite should be compared with the fracture toughness values of the epoxy (1 MPa.m$^{1/2}$).

Terminology

- **Piece**: The finished product that you are making, a kayak or sailboard, for example.
- **Mould**: The thing from which the piece is fabricated. The mould from which you would fabricate a kayak will look very much like a kayak except that it will be smooth on the inside. The inside of the mould will be duplicated on the outside of the piece.
- **Plug**: When the mould itself is constructed from fibreglass, the starting point is the plug. The mould is constructed from the plug. The plug, used to construct a kayak, will look exactly like a kayak on the outside. However, the plug can be constructed from anything so long as it has a smooth non-porous surface.
- **Laminate**: A solid constructed of successive plys of resin and fibreglass.
- **Ply**: A layer of cloth impregnated with resin and allowed to cure (set). Most structures are fabricated from several plies, the plies may have the same or(more usually) different orientations.
- **Gel Coat**: The outer-most surface of the mould or piece, its purpose is cosmetic. It gives the piece and the mould the colouration, finish and durability desired. The gel coat consists of a thin layer of specially compounded resin.
- **Mould Release**: The material used to effect a release between a mould and the moulded part.
- **Cure**: The process in which the liquid resin becomes a solid and bonds to the fibres. This is a chemical reaction between eith different components of the resin or between the resin and a hardner. The cure can be effect by heat or the addition of a catalyst.

Fundamentals of Polyester Resin systems, Including Gel Coats

The curing mechanism of polyester resin is inhibited by air. Therefore if the resin is exposed to air for a considerable time, the surface may remain sticky for several weeks. If the coat is cured rapidly, as in a thick laminate, inhibition will not occur because of the heat generated by the curing reaction. However, when applying thin films of polyester resins, such as surface coats, it is necessary to add styrene wax to the resin to obtain a full cure. The wax rises to the surface during the cure and can be later removed with soap and water.

Wax does not need to be added to gel coats applied against the mould as subsequent layers prohibit air inhibition. Wax is only necessary when the gel coat is exposed to air.

Building The Plug

The plug may be constructed of nearly anything. It is the surface finish that is important. Quite often it is desirable to merely reproduce, in fibreglass, an item already on hand. In such a case the item already on hand will serve as the plug. In most cases, the plug is constructed from scratch from some easy to work with medium. An ideal medium is styrofoam or polyurethane. In these cases the plug must be coated with an epoxy resin rather than a polyester resin (just watch the styrofoam dissolve). After sanding the plug smooth coat with a thin layer of resin and cover with a layer of 35gm/sq.m

(1oz/sq.yd) glass fibre cloth. After drying brush on several thin layers of resin, sanding between layers. **The surface finish at this point is the most important**.

Mould Release

Mould release must be applied to the plug - this is an important step in the process. If the release agents fail to perform the mould will not release from the plug and many hours will be required to fix the damage and develop a smooth surface with the desired geometry. The usual method of applying mould release is 3 layers of carnuba wax. Each layer should be left to dry fully 1-2 hours then buffed to a shine. This is followed by a light coat of PVA film either sprayed or brushed on. Allow to dry overnight before applying the gel coat.

Gel Coat

The gel coat is the first step in making the actual mould. The gel coat should be applied in two coats of 0.4mm (.015") each, allowing 1-2 hours between coats. The gel coat should be tacky but not wet before proceeding. Use MEKP (MethylEthylKetone Peroxide) hardener at 2% by volume with polyEster gelcoats.

Gel Coat Troubleshooting

Application of the gel coat is perhaps, the most difficult aspect of manufacturing in fibreglass. Problems arise due to temperature variation, catalysing and a variety of handling techniques. The following list should help resolve some of the difficulties.

- **Wrinkles and Pinholes**
  A coating less than (0.005") thick may wrinkle especially when brush marks are present. The preferred thickness is 0.25mm (0.010") to 0.5mm (0.020"). A wrinkle can also occur if the gel coat is not cured enough prior to lay-up. Before lay-up check the surface for tack. The surface should be sticky but not transfer to your finger. Several things can affect a slow cure; solvent or water entrapment, undercatalysation and cold temperatures. If the gel coat contains pinholes, check the spray equipment for moisture in the air lines or dirt in the traps.
- **Slow Gel Coat Cure**
  Any temperature below 21°C (70°F) will retard the gel time. An under-catalysed gel coat will result in a slow cure but this is not a recommended technique for deliberately lengthening gel time. The usually ratio of initiator (MEKP) to Polyester Resin is 2% by volume. High moisture and humidity will lengthen gel time.
- **Sagging of the Gel Coat**
  The most common reason for the gel coat to sag is because the application was too heavy. To avoid this, spray in multiple light applications from about 15" distance from the mould. The maximum application to be sprayed at on application is 0.4mm (.015").
- **Porosity**
  Gel coat porosity (air bubbles) is normally caused by trapped air or moisture. This can be prevented by limiting the gel coat application to about 0.1mm (.005") per pass. Avoid excessive air pressure, 40 -80 psi is adequate depending on resin viscosity.
- **Blisters When Immersed in Water**
  Blisters are caused by an incomplete cure, improper wetting of the backing fibres, a bad bond between the gel coat and the back up laminate (Usually due to grease) or too thin a gel coat.
- **Lifting of the Gel Coat before Laminating**
  This is caused by shrinkage of the gel coat because the coat cures too fast because of over

catalysation, delays between application of the gel coat and laminating or the mould surface is too hot (usually the mould has been in direct sun light and is over heated.

- **Back up Pattern Showing**
  Gel Coat is too thin or was not sufficiently cured before laminating
- **Gel Coat sticking to the Mould/Part**
  This condition is caused by improper releasing of the mould surface. Must use release agents developed for the fibreglass industry.

Laminating

Once the final layer of gel coat has been allowed to dry for several hours the skin lamination may be applied. For making the mould use a lightweight glass cloth, about 25gm/sq.m (0.75 oz/sq.yd). Apply a coat of moulding resin and gently lay the mat into the resin. Careful cutting of the cloth may be necessary to ensure conformance to the surface of the plug. Use a stiff bristle brush to force resin into the cloth and to make it conform to the various contours of the plug. Dabbing is better than brushing, A short mohair roller is good for long gently curving surfaces. All air must be worked out so that the matt is tight against the plug surface. Allow 2-3 hours to dry before proceeding with the next layer of the lamination using a 50gm/sq.m (1.5 oz/sq.yd)cloth. This should be followed up immediately with a 350gm/sq.m (10oz/sq.yd). fabric. The mould should be built up such that its final thickness is about 2 to 3 times that of the parts that are to be produced from it. One the heavy fabric layer has been allowed to cure for about 3 hours a combination of woven roving or chopped strand mat under a 50 gm/sq.m (1.5oz/sq.yd) cloth can be added to build up the required thickness. Use MEKP hardener at 1% by volume with polyEster moulding resin.

Removing the Mould

Allow the completed mould to cure for 48 hours before attempting to remove it from the mould. Care and time are required to get the mould off the plug. Soft plastic wedges can be driven in between the plug and mould to facilitate the release. Once free of the plug, replace the mould on the plug and add bracing to the mould using wood or metal to prevent distortion of the mould. Wait 24 hours before removing from the plug. The mould edges should be trimmed and the PVA washed out with warm soapy water. Any wrinkles or air bubbles in the gel coat should be filled and sanded plug to the mould at this time.

Moulding the Piece

Once the mould has cured for at least 72 hours you are ready to apply the mould release wax and PVA parting film exactly as was done when preparing the plug. Apply one 0.5mm (.020") layer of pigmented gel coat as before, waiting the 2 to 3 hours before continuing with the laminations, employing the same technique and care as used in preparation of the mould.

Safety Considerations

Styrene, the solvent and cross-linking monomer used in polyester resins is a highly volatile and dangerous substance, being both flammable and toxic when inhaled. Styrene will act as a solvent for the fatty tisues of the lungs and results in liquation (drowning) of the interior of the lungs - symptoms are similar to tuberculosis. Work in a well ventilated area only - the great outdoors is perhaps best - but avoid direct sunlight. Use a respirator capable of absorbing organic vapours. Do not have any open flames anywhere near the resin as combustion will be instantaneous. Remember that the temperature must remain above 20°C (75°F) for 24 hours to allow for proper curing. If you must work indoors use a bagging system to contain the vapour during curing.

Multiple Piece Moulds

..... to be continued.

Fundamentals of Epoxy Resin Systems

..... to be continued

Fibreglassing Polystyrene Foam

The two most common types of foam are expanded or pre-expanded polystyrene foam. Expanded polystyrene foam is commonly used in the building industry as an insulation material while pre-expanded foam is used as a packaging material. It is often desirable to apply a skin of durable fibreglass to these foams, however, **both are dissolved by the more common and inexpensive polyester resins** and as such the more expensive epoxy resins must be used instead.

Materials

Fibreglass cloth comes in many varieties. The most commonly used in skin laminations are:

Epoxy resin is a two component, low viscosity epoxy system for applying a skin lamination. The normal mixing ratio, #88 Epoxy resin to #87 Epoxy hardener is 5:1 epoxy to hardener. Colour pigments can be added to the epoxy resin system. The best way to apply the resin is to use a short nap paint roller for both the priming coat and saturating the cloth, finish coats are best applied with a good quality nylon paint brush.

Methods

- The temperature must be above 23°C (75°F) for at least 24 hours after application.
- Work in a well ventilated and dust free environment - avoid direct sunlight.
- Always add the hardener to the resin and mix well before applying.
- The surface to be fibreglassed should be clean and dust free - a fine grit sandpaper will suffice. Do not use chemical cleaners other than household detergent as they will attack the foam.

Prime Coat

A litre of resin will cover about 3.5 sq.m (35 sq. ft). The initial application should be thick, but not too thick as to cause runs or sags. Allow to dry for several hours before applying the glass. Resin should be mixed in small quantities e.g. 100ml of hardener to 0.5 litre of epoxy resin. This will last about 15 minutes before it hardens.

Glass Cloth

Cut the cloth so that it more or less conforms to the surface to be covered. Final trimming should be done while saturating the cloth. Apply a thin coat of the mixed resin to the surface and immediately cover with the glass cloth. Smooth out the glass cloth. Apply additional resin to the glass cloth so that it is fully saturated and transparent. Make sure there are no blisters, air pockets or dry areas. If more than one piece of glass is used, make sure each is fully saturated before proceeding to the next piece - alloy about 2.5cm (1") overlap between adjacent pieces.

Allow 24 hours to cure. Sand out any laps, seams or other surface regularities with a 100 to 220 grit sandpaper.

If a colour pigment is to be added to the final coat it is a good idea to slightly tint the glass layer. use about 1 teaspoon (5ml) of pigment per 250ml of resin.

Finish Coat

If colour pigment is used add at the rate of 1 teaspoon (5ml) to 250ml of resin. Mix all the resin you will need at one time to avoid any variations in coloration. Thixotropic silica can be added to prevent sags and runs at the rate of 75ml of silica to 250ml of epoxy resin - allow about 10 minutes to thicken - before adding the hardener.

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16 . Properties of Selected Fibres

	Density (gcm^{-3})	Longitudinal Tensile Modulus E_1 (GPa)	Transverse Tensile Modulus E_2 (GPa)	Poisson's ratio □$_{12}$	Shear Modulus G_{12} (GPa)	Longitudinal Tensile Strength □ (MPa)	Longitudinal Thermal Expansion □$_1$ (10^{-6} K^{-1})	Transverse Thermal Expansion □$_2$ (10^{-6} K^{-1})	Heat Resistance °C	Cost $/kg
Glass	2.45	71	71	0.22	30	3500	5	5		0.70
PBT	1.58	365				5800				15.00
Kevlar (49) Kevlar (29)	1.47	154 61	4.2	0.35	2.9	2800	-4	54	550 450	4.50
PE (Spectra) PE (Dyneem	0.97 0.975	66-124^{1} 115				2300-3250^{2} 3500	- -12		150	12.00

a)										
PBO Zylon AS PBO Zylon HM	1.54 1.56	180 270				5800	-6		650	15.00
Graphite (AS)	1.75	224	14	0.2	14	2100	-1	10		7.00
Graphite (HMS)	1.94	385	6.3	0.2	7.7	1750	-1	10		8.00
Boron	2.45	420	420	0.2	170	3500	5	5		54.00
SiC	3.2	406	406	0.2	169	3395	5.2	5.2		75.00
Saffil (5%SiO_2-Al_2O_3)	3.3	300	300	0.2	126	1500	5.2	5.2		2.50
Al_2O_3	3.9	385	385	0.3	154	1400	8.5	8.5		25.00

[1] Spectra 900 E=66-73GPa; Spectra 1000 E=98-113GPa; Spectra 2000 E=113-124GPa

[2] Spectra 900 □=2.1-2.6GPa; Spectra 1000 □=2.9-3.25GPa; Spectra 2000 □=2.9-3.5GPa

PBT = Poly(p-phenylene-2,6-benzobisthiazole)

PE = Gel Spun ultra high molecular weight polyEthylene (Spectra®,Dyneema®)

PEN = polyEthylene Napthalate (Pentex®

PBO = Poly(p-phenylene-2,6-benzobisoxazole)

Properties of Seleced Matrices

	Density (gcm^{-3})	Tensile Modulus E (GPa)	Poisson's ratio □	Shear Mudulus G (GPa)	Longitudinal Tensile Strength □ (MPa)	Longitudinal Thermal Expansion □ (10^{-6} K^{-1})	Glass transition melting Point (°C)	Cost ($/kg)
Epoxy	1.54	3.5	0.33	1.25	60	57.5		8.00

PolyEster	1.38	2.5	0.33	1.2	35		260	2.00
polyVinylEster	1.5	3.0			60		260 (T_{max})	3.00
PolySulfone (pS)	1.25	2.7			106	56	190 (T_g)	14.00
PolyPhenyleneSulfone (pPS)	1.25	4.8			135	56	93 (T_g) 285 (T_m)	14.00
PolyEtherSulfone (pES)	1.37	2.6			129	55	230 (T_g)	14.00
PolyEtherImide (pEI)	1.27	3.3			145	62	217 (T_g)	17.00
PolyAmideImide (pAI)	1.4	4.8			197	63		16.00
polyAmide (nylon)	1.36	2.5			100			6.00
PEEK	1.3	3.6	0.37	1.4	70	45	143 (T_g) 334 (T_m)	18.00
Aluminium	2.71	69	0.32	26	74*	23.6	660	2.00
Titanium	4.51	113.8	0.33		238**	8.4	1670	18.00
Magnesium	1.74	45.5	0.33	7.5	189	26	650	4.00
Borosilicate glass	2.23	63.7	0.21	28	90	3.25		0.50

17

*** Aluminium Alloys such as 2024, 6061 and 7075 can be used as a matrix and can be heat treated to give tensile strengths up to 550 MPa.**
**** Titanium alloys such as IMI 550, Ti-6Al-4V can be used as matrices and have yield strengths in the range 800 to 1500 MPa.**

For shear modulus a good approximation is 3/8 of Elastic modulus. Polymer Matrices

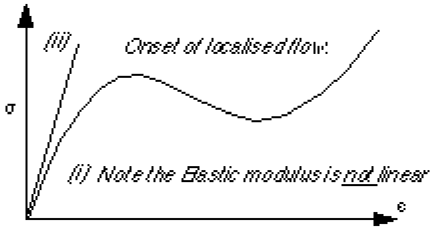
In examining potential composites systems both the manufacturing route and end use need to be considered. Like metals we need to consider two stock materials, cast and wrought. Mouldings can be of two types, one where the fibres are prepositioned and the resin - generally a thermoset, (though there is no reason why a thermoplastic could not be used) is injected into the preform - mould and allowed to cure (Resin Transfer Moulding) or where the two constituents of the resin (one or both containing short fibres) are rapidly mixed and injected into the mould (Reaction Injection Moulding). Wrought products such as bar and tube can be fabricated by pultrusion, while shell structures can be fabricated by diaphragm forming. In both cases the feed stock is in the form of pre-impregnated fibres, where the polymer is precoated onto the fibres before they are wound into tape (unidirectional) or woven into mat.

Material	E, GPa	UTS, MPa	K, $MNm^{-3/2}$	T_{glass}, °C	T_{melt}, °C	T_{max}, °C
PVE	1.5 to 4	40 to 80	1 to 3	-	-	260
PEI	3.3	93 to 197	10	200	217	170
PAI	2.5	70 to 100	2.2	220	355	260
PEEK	3.6	90 to 170	7.5	143	334	250
PPS	4.8	80 to 150	2	93	285	240
POLYESTER	1-4	30-70	0.5	-	-	-
Epoxy	2-5	40-80	1	-	-	-
Nylon pA	2-3	60-120	3-5	-	-	-

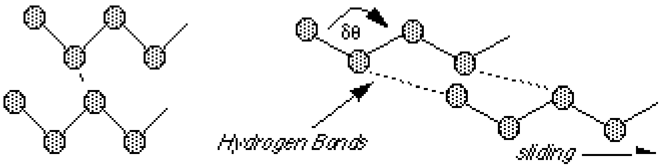
Polymer	abbr.	type	Manufacturer
poly(Vinyl Ester)	PVE	Thermoset	(Dow Chemical)
poly(AmideImide)	PAI	Thermoplastic	(TORLON - Amoco Chemicals)
poly(EtherImide)	PEI	Thermoplastic	(ULTEM - General Electric Plastics)
poly(EtherEtherKetone)	PEEK	Thermoplastic	(VICTREX PEEK - ICI)
poly(phenyleneSulphide)	PPS	Thermoplastic	(Philips Petroleum)

In order to chose suitable polymer matrices we need to understand what - at the molecular level - is responsible for for the mechanical properties such as elasticity and strength. We must also be aware of the difference between thermosetting polymers and thermoplastic polymers and the glass transition temperature.

A schematic stress -strain plot for an amorphous thermoplastic above (i) and below (ii) its glass transition temperature.



The polymer may be visualised as a number of coils of rope which are tangled together. The chains may or may not contain polorisable groups such as N-H or O-H bonds and may or may not contain side groupings which distort the linearity of the molecules. As the polymer is loaded the interchain bonds stretch and the moleculs slide past each other, trying to uncoil - a process known as reptition. The stronger the interchain bonds, the stiffer the polymer. Thermosetting polymers, in which the chains are bound together by C-C bonds are generally stiffer than thermoplastics in which the interchain bonds are Hydrogen bonds. Nonetheless the stiffness is low because the C-C bonds in the chains are not aligned along the testing direction and are more likely to act as a stiff hinge.



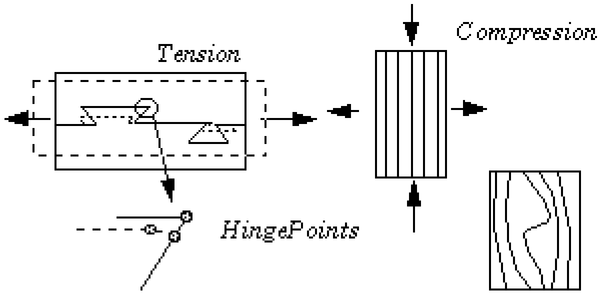
Below the glass transition temperature the interchain bonds limit motion of the chains, above T_g the bonds effectively melt leaving the polymer chain s free to move past each other giving a more 'plastic' or viscoelastic response. For thermosets and thermoplastics below Tg releasing the stress will cause the molecules to spring back to their original positions - If the stress is large enough to cause yield then yielding will be limited. Plastic deformation is easiest in the simplest molecules and impossible in highly cross-linked polymers. Yielding can occur by **SHEAR BANDING** or **CRAZING**.

Shear Bands

Shear bands are regions of intense local deformation parallel to the direction of maximum shear - it may be possible for the shear bands to align parallel to the applied stress creating a hig strength neck and for a period of extended necking/ductility to occur as the polymer draws out. In the complex polymers this is not the case and rarely happens in thermosetting polymers. The yield criteria is best represented by a Von Mises criterion

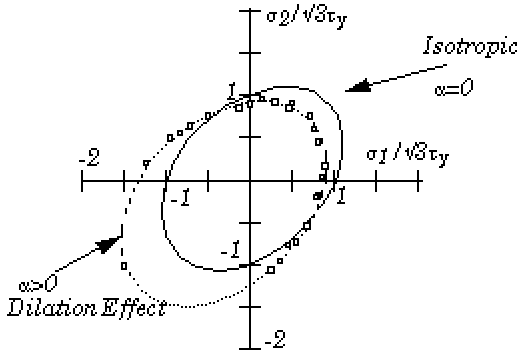
$$\frac{1}{\sqrt{6}}\left((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right)^{1/2} \geq \tau_y$$

Just to complicate things, the shearing process is relatively simple in tension where there is a dilattory component in the applied stress, ie. the mean stress is positive, but more difficult in compression:



To account for this problem the yield criterion is modified and the polymer is usually stronger in compression than in tension for biaxial tension this reduces to.

$$\sigma_1^2 + \sigma_1\sigma_2 + \sigma_2^2 \geq 3\tau_y - \alpha\sigma_m$$



which delays yielding to a higher stress when the mean stress is compressive. This equation is elliptical when a is zero and distorted when a is positive.

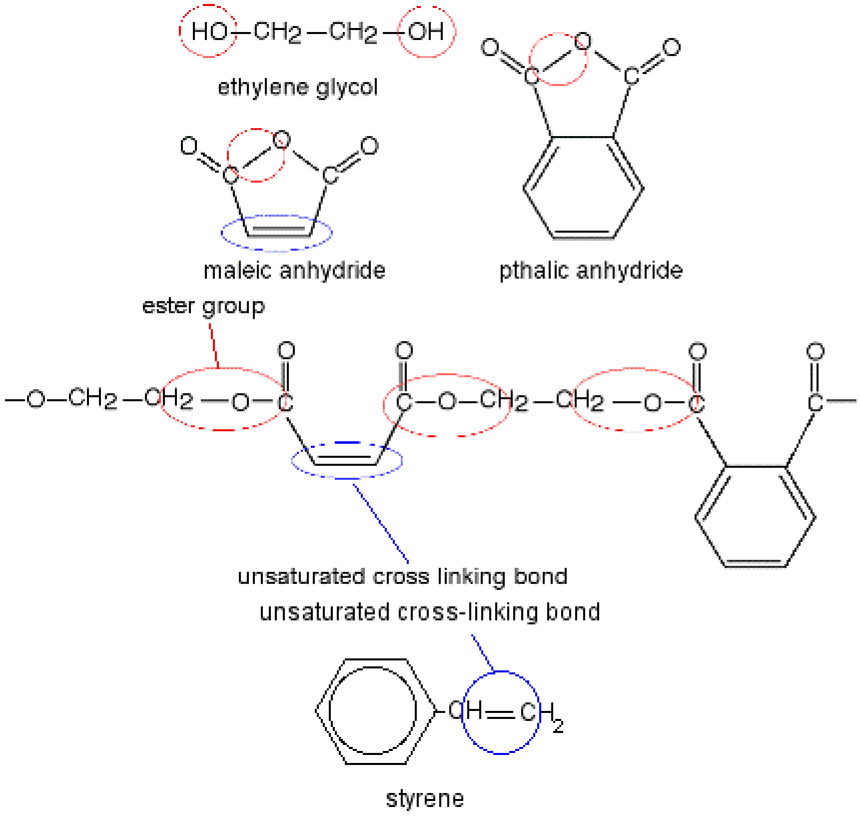
Crazing

Crazing is an alternative and competitive mechanism of permanent deformation in polymeric materials. Crazing becomes more favourable as the temperature is lowered. However, since the crazing process produces a hole in the polymer, ie. an increase in volume, it cannot occur under compression. Crazes appear like cracks whichrun perpendicular to the applied stress.

The craze is basically a narrow crack that is bridged at intervals across its surface by fibrils of polymer chains. The chains are highly drawn (aligned) in the direction of the maximum principal stress and serve to keep the craze crack from opening. The craze extends with little increase in thickness. Overall ductility is low as fracture occurs by the breakage of the fibrils and the macroscopic extension of the crack. (note: deformation within the craze itself is very large). Crazing occurs based on the magnitude of s1-s2>0 ie. a strain based criterion.

Thermosetting Polymer Matrix - Fibre Composites

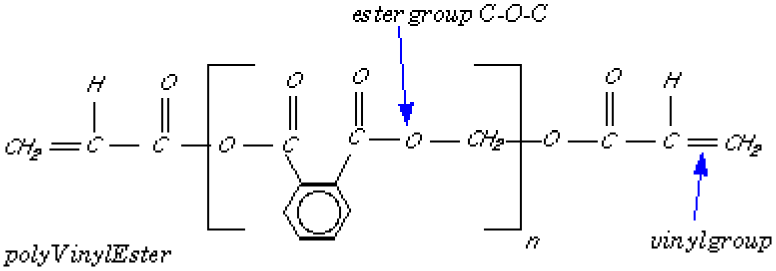
There are two types of thermosetting matrix, polyester and epoxy, both of which have been reinforced with glass, carbon and kevlar fibres. Polyester is the simplest of the thermosets and is generally made by reacting dibasic acids such as maleic anhydride or phthalic anhydride with dihydric alcohols such as ethylene glycol (antifreeze) in approximately equal amounts. The resulting polymer is a short chain polymer with a molecular weight of about 5000 (about 35-40 repeat units or mers) and is a stable liquid. The liquid resin is set into an amorphous solid by cross linking the polyester chains to each other. The cross linking occurs by the addition of a small monomer molecule such as styrene. The monomer, like the active sites on the polyester chain has an unsaturated C=C bond and it is this that provides the bridge between the polyester chains.



The monomer is usually mixed with the polyester to form 'fibreglass' resin. The two components will react but at room temperature the reaction may take years and needs to be initiated by the addition of a catalyst (loosely referred to as a hardener) such as methyl ethyl ketone epoxide at about 0.5 to 1% by volume. Once the reaction is initiated, the cross linking generates sufficient heat to further catalyse the reaction and the resin gels (within about 10 minutes) the cross linking continues and full strength is generated after about 24 hours.

Chemical	mole fraction	kg/100kg resin
phthlalic anhydride	0.2	28.86
maleic anhydride	0.15	19.11
propylene glycol	0.2	14.83
ethylene glycol	0.15	12.10
styrene	0.3	30.00

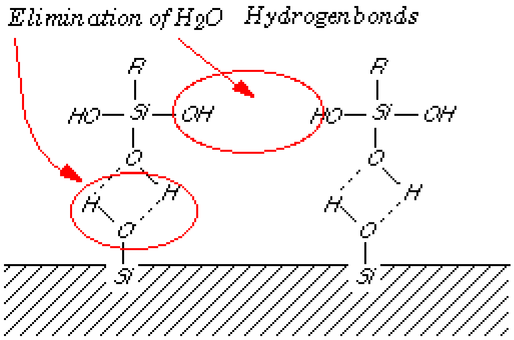
Vinyl Ester resins differ from ester resins in that the ester resin has one unsaturated group per mer unit, i.e. about 30 to 40 cross linking sites per polyEster molecule while the vinyl ester has only two unsaturated groups, one at each end of the molecule. Polyesters, being more cross-linked are more brittle than their vinyl ester counterparts.



Resin-Fibre Interface (Coupling agents) for Glass/Polyester

In order for the composite to function properly there must be a chemical bond between the matrix and the re-inforcing fibres in order that the applied load (applied to the matrix) can be transferred to the fibres (which are expected to do all the work. However, the bond must not be too strong since the toughness of the composite comes from such sources as fibre pullout and fibre-matrix interfacial fracture. In 'fibre glass' the fibre is inorganic while the matrix is organic and the two do not bond readily unless the fibres are treated to modify their surface.

Silica (SiO_2) is hygroscopic ie. it absorbs water onto its surface where the water breaks down into hydoxyl (-OH) groups. It is impossible to avoid the water especially as the surface modifier or ‘size' is applied in a water based solvent. It should also be stressed that water reduces the strength of SiO_2 by a stress-corrosion-cracking mechanism. The coupling agent takes the form of a silane ($R\text{-}SiX_3$) where R is an organic radical that is compatible with the polymer matrix (it may even react with the matrix polymer; for this reason styrene groups are favoured for polyesters while amine groups are preferred for epoxies) and X is a hydrolisable organic group such as an alcohol. The most common silane couplant is tri-ethoxy-silane.Heat will force the elimination of water between the -OH pairs at the hydrated silica surface and the silane as well as between the adjacent silane moecules.

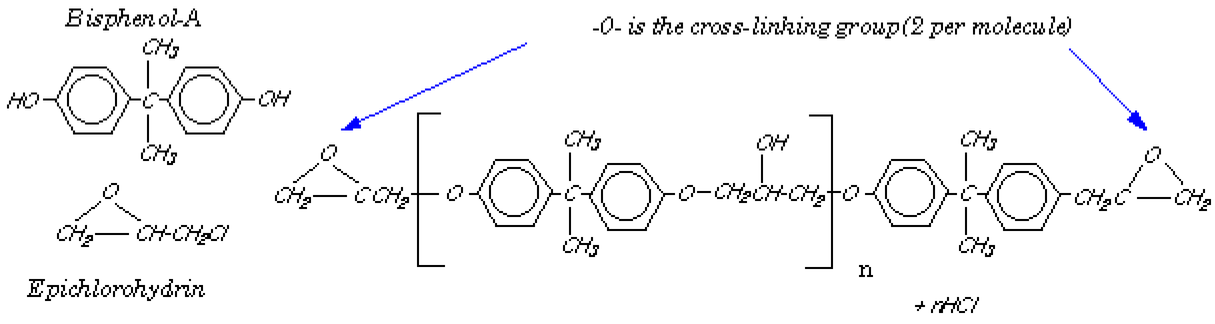


Epoxy Resins

Epoxy resins are much more expensive than polyester resins because of the high cost of the precursor chemicals most notably epichlorohydrin. However, the increased complexity of the 'epoxy' polymer chain and the potential for a greater degree of control of the cross linking process gives a much improved matrix in terms of strength and ductility. Most epoxies require the resin and hardner to be mixed in equal proportions and for full strength require heating to complete the curing process. This can be advantageous as the resin can be applied directly to the fibres and curing need only take place at the time of manufacture. - known as pre-preg or pre impregnated fibre.

Epoxy polymers are made by reacting epichlorohydrin with bisphenol-A in an alkaline solution which absorbs the HCl released during the condensation polymerisation reaction.

Each chain has a molecular weight between 900 and 3000 (about 3 to 10 mers) with an epoxide grouping at each end of the chain but none within the polymer chain. The epoxy is cured by adding a hardner in equal amounts and being heated to about 120°C. The hardners are usually short chain diamines such as ethylene diamine. Heat is usually required since the cross linking involves the condensation of water which must be removed in the vapour phase.



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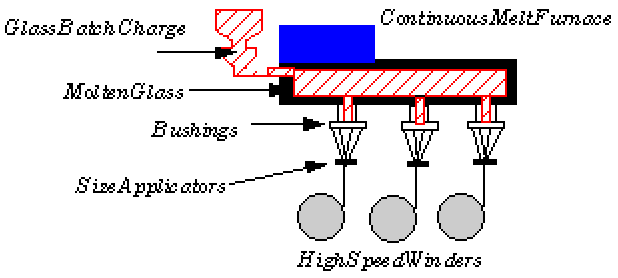
18 Glass Fibres

In order to handle the fine filamentary fibres that are necessary for structural composites the fibres are usually in the form of bundles. The bundles are drawn continuously from platinum-rhodium bushings, each producing several hundred filaments. The fibres are pulled away at speeds approaching 1000 to 2000 m/min. as molten glass and coated with size which lubricates the surface to prevent abrasion before the filaments (100-1200) are brought together into a tow and wound onto a mandrel.



The Glass fibres are available in several forms. The main varieties are:

1. Chopped Strands - Short lengths of fibre in bundles of ~100fibres. Length 3 to 40 mm used in automated pressing and mouldingwith both thermosetting and thermoplastic matrices as well ascement.
2. Chopped Strand Mat - chopped strands in the size 30 to 40mmare distributed over the area of a conveyor belt in randomorientations an a small amount of an organic binder (usually polyvinyl acetate) added to form a loosely bound open mat which isreadily impregnated by resin. Binder must be compatible with theresin - normally used with polyester or epoxy resins.
3. Rovings - A number of strands are grouped together and woundwithout twisting onto a cylindrical package to give a longcontinuous rope or large tow that may be used for filament windingor for chopping and spraying.
4. Yarn - twisted strands used in weaving cloth.

The glass comprises one of two main types: Traditional glass fibre used for reinforcing rubbers and plastics and Alkali resistant for reinforcing cement.

Types of Glass

- E Glass is the all purpose fibre. Resistant to leaching inwater.
- C-Glass is acid resistant - not generally used as areinforcement.
- A Glass is a typical window glass for comparison purposes andis not used in fibre manufacture
- S Glass has highest strength and stiffness
- AR glass is alkali resistant and used for strengtheningcements.

Fibre	SiO_2	Al_2O_3	Fe_2O_3	B_2O_3	ZrO_2	MgO	CaO	Na_2O	K_2O	TiO_2
A-Glass	73	1	0.1	-	-	4	8	13	0.5	-
E-Glass	52-56	12-16	0-0.5	8-13	-	0-6	16-25	-	-	-
AR Glass	60-70	0-5	-	-	15-20	-	0-10	10-15	-	0-5
C Glass	59-64	3-5	0.1-0.3	6-7	-	2-4	13-14	8-11	0.4-0.7	-
S Glass	50-85	10-35	-	-	-	4-25	-	0		-

Fibre	E(GPa)	Strength (MPa)	□	Density (g/cm^3)	T_m or T_g (°C)
A-Glass					
E-Glass	72	1700 - 2100	0.2	2.50	1500
AR Glass	70	1500 - 1900			
C Glass					
S Glass	80	2000 - 2500	0.22	2.54	1400

19 . Carbon Fibres, Carbon-Polymer Composites and Carbon-Carbon Composites

Carbon-Polymer composites consist of carbon fibres, either unidirectional, woven or knitted or 3-Dimensional embedded in a polymer matrix. The polymer matrix can be either a thermosetting resin such as epoxy or a thermoplastic resin such as PEEK. While woven carbon fibre is available un-coated for use in wet lay up manufacturing, the majority of carbon fibre is pre-coated or pre-impregnated (pre-preg) with the polymer. Carbon-carbon composites consist of highly-ordered graphite fibers embedded in a carbon matrix. C-C composites are made by gradually building up a carbon matrix on a fiber preform through a series of impregnation and pyrolysis steps or chemical vapor deposition. C-C composites tend to be stiffer, stronger and lighter than steel or other metals.

PRODUCTION PROCESSES - Carbon Fibres

The production of carbon based composites begins with the production of the carbon fibers. Carbon fibers are long bundles of linked graphite plates, forming a crystal structure layered parallel to the fiber axis. This crystal structure makes the fibers highly anisotropic, with an elastic modulus of up to 5000GPa on-axis versus only 35GPa off-axis. Fibers can be made from several different precursor materials, and the method of production is essentially the same for each precursor: a polymer fiber undergoes pyrolysis under well-controlled heat, timing and atmospheric conditions, and at some point in the process it is subjected to tension. The resulting fiber can have a wide range of properties, based on the orientation, spacing, and size of the graphite chains produced by varying these process conditions.

Precursor material is drawn or spun into a thin filament. The filament is then heated slowly in air to stabilize it and prevent it from melting at the high temperatures used in the following steps. The stabilized fiber is placed in an inert atmosphere and heated to approximately 1500°C to drive out the non-carbon constituents of the precursor material. This pyrolysis process, known as carbonization, changes the fiber from a bundle of polymer chains into a bundle of "ribbons" of linked hexagonal graphite plates, oriented somewhat randomly through the fiber. The length of the ribbons can be increased and their axial orientation improved through further heating steps up to 3000°C, a process called graphitization. Because the graphite ribbons are bonded to each other perpendicular to the fibres only by weak Van der Waals bonds, the ribbons must be reoriented to increase the tensile strength of the fiber to a useful level. This is accomplished through the application of tension at some point in the stabilization or pyrolysis phases, the exact time depending on the precursor material. Increased axial orientation increases the fiber's tensile strength by making better use of the strong covalent bonds along the ribbons of graphite plates.

There are three principal precursor materials for carbon fibers, of which **polyacrylonitrile (PAN)** and **rayon** are the most common. PAN is stretched during the stabilization phase, and heated to 250°C in air. The tension is then removed, and the fiber is heated slowly in an inert nitrogen atmosphere to 1000-1500°C. Slow heating maintains the molecular ordering applied by tension during the stabilization phase. Graphitization at temperatures up to 3000°C then follows. Applying tension at 2000°C further increases the proper ordering of graphite ribbons. Rayon, a cellulose-based fiber made from wood pulp, is spun into a filament from a melt, and stabilized without tension up to 400°C. It is then carbonized without tension up to 1500°C, and is stretched in the graphitization phase up to 2500°C. **Pitch**, as PVC, coal tar or petroleum asphalt, is also used as a precursor. The pitch material is spun into a filament from either a simple melt or from a liquid crystal "mesophase" melt (obtained by heating the melt above 350°C for an extended period). If the precursor filament is obtained from the simple melt, it must have tension applied during the graphitization phase. When a mesophase melt is used, the spinning process imparts a high degree of order to the resulting graphite, so tension need not be applied during the pyrolysis phases. Filaments from pitch are otherwise processed into fibers by the same process as other precursors. Polyvinyl alcohol, polyamides and phenolics may also be used as precursors. Precursors must be able to be carbonized without melting.

Manufacturer Trade Name	Grade	Density (gcm^{-3})	Stiffness (GPa)	Strength (GPa)	Strain at Failure (%)
BASF Celion	G30	1.78	234	3.79	1.62
	G40	1.77	300	4.97	1.66
	G50	1.78	358	2.48	0.7
Hercules Magnamite	AS4	1.8	235	3.8	1.53
	IM6	1.73	276	4.38	1.50
	HMU	1.84	380	2.76	0.7
Amoco Thornel	T-650/35	1.77	241	4.55	1.75
	T-650/42	1.78	290	5.03	1.70
	T-50	1.81	390	2.42	0.7

Properties of PAN-based carbon fibres - available in 6000 or 12000 fibres/tow

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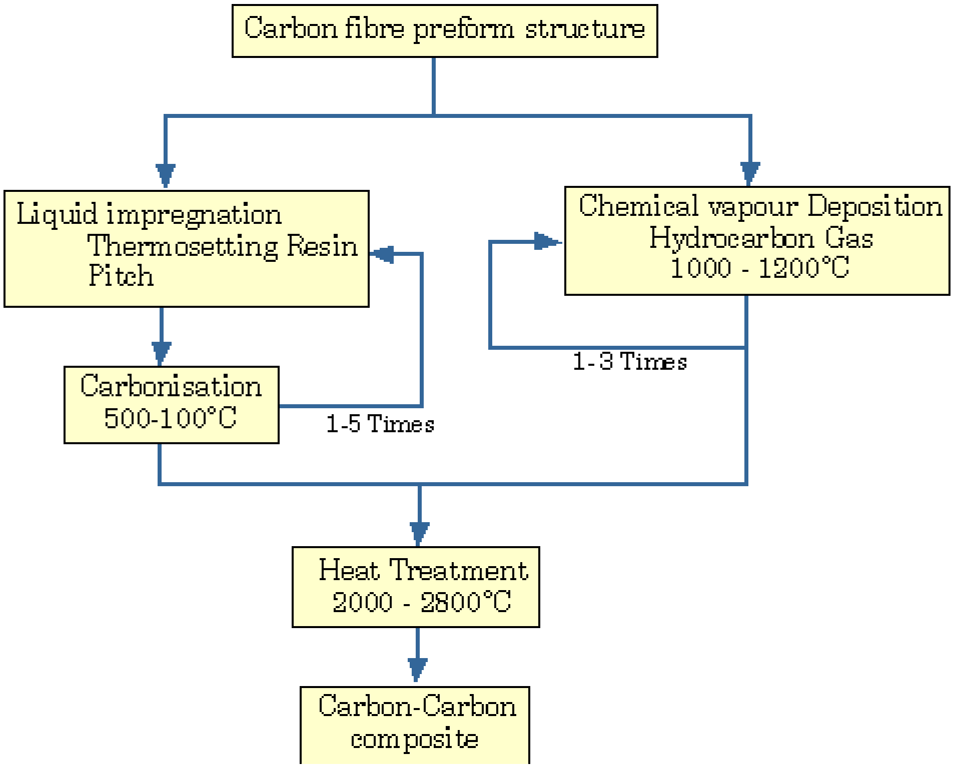
PRODUCTION PROCESSES - Carbon-Polymer Composites

- Wet Lay-up
- Pre-preg (Epoxy)
- Pre-preg (Thermoplastic)
- Pultrusion
- Filament Winding
- Diaphragm Forming
- Injection Moulding

PRODUCTION PROCESSES - Carbon - Matrix



The second phase in carbon-carbon composite production is the building up of the carbon matrix around the graphite fibers. There are two common ways to create the matrix: through chemical vapor deposition and through the application of a resin.

Chemical vapor deposition (CVD) begins with a preform in the desired shape of the part, usually formed from several layers of woven carbon fabric. The preform is heated in a furnace pressurized with an organic gas, such as methane, acetylene or benzene. Under high heat and pressure, the gas decomposes and deposits a layer of carbon onto the carbon fibers. The gas must diffuse through the entire preform to make a uniform matrix, so the process is very slow, often requiring several weeks and several processing steps to make a single part. In the second method a thermosetting resin such as epoxy or phenolic is applied under pressure to the preform, which is then pyrolized into carbon at high temperature. Alternatively, a preform can be built up from **resin-impregnated** carbon textiles (woven or non-woven) or yarns, then cured and pyrolized. Shrinkage in the resin during carbonization results in tiny cracks in the matrix and a reduction in density. The part must then be re-injected and pyrolized several times (up to a dozen cycles) to fill in the small cracks and to achieve the desired density. Densification can also be accomplished using CVD.

A limiting factor on the use of carbon-carbon composites is the manufacturing expense associated with these slow and complex conventional methods. In response, two less-expensive alternative methods for building up the carbon matrix have been developed. The first is a forced-flow/thermal gradient process developed at the Georgia Institute of Technology in Atlanta, and is a variation on CVD. This method deposits carbon matrix up to 30% faster than conventional methods, and allows thicker items to be produced. Carbon-bearing propylene, propane or methane is forced under pressure through the preform while it is heated in an oven at 1200°C. A temperature gradient in the material forces vapor to flow through the preform, ensuring the even formation of the matrix. Vapor infiltration and carbon deposition are faster with this method, so parts up to 1cm thick can be produced in as little as eight hours. Parts up to 2cm thick (with material properties comparable to

CVD-produced parts) have also been produced. Because the process itself ensures uniform vapor infiltration, it can be run under a wider range of operating conditions than CVD -- the process is less dependent on precise heating, pressure and timing conditions. In the future, this flexibility may even allow the addition of graphitization catalysts and oxidation preventers during production, thereby eliminating a separate treatment. T he second alternative method was developed by the Across Company of Japan, and is a variation on pre-impregnated or "pre-preg" materials used to create a preform. Graphite yarns are coated with graphite precursor powders made from coke and pitch, and are then sealed in a flexible thermoplastic sleeve to protect the powder coating during handling and manufacture. The treated yarn can then be woven into sheets or chopped into short fibers and applied to a mold. The laid-up form is then hot-pressed to make the composite part. Yarns can also be processed into tubes, rods, cloth, thick textiles, unidirectional sheets and tapes. Better penetration of the matrix into fiber bundles ensures uniform properties in the composite and higher strength than conventional composites. Fewer densification steps are needed, so manufacturing time and costs are reduced.

GENERAL PROPERTIES

The most important class of properties of carbon-carbon composites is their thermal properties. C-C composites have very low thermal expansion coefficients, making them dimensionally stable at a wide range of temperatures, and they have high thermal conductivity. C-C composites retain mechanical properties even at temperatures (in non-oxidizing atmospheres) above 2000°C. They are also highly resistant to thermal shock, or fracture due to rapid and extreme changes in temperature. The material properties of a carbon-carbon composite vary depending on the fiber fraction, fiber type selected, textile weave type and similar factors, and the individual properties of the fibers and matrix material. Fiber properties depend on precursor material, production process, degree of graphitization and orientation, etc. The tensioning step in fiber formation is critical in making a fiber (and therefore a composite) with any useful strength at all. Matrix precursor material and manufacturing method have a significant impact on composite strength. Sufficient and uniform densification is necessary for a strong composite. Generally, the elastic modulus is very high, from 15-20GPa for composites made with a 3D fiber felt to 150-200GPa for those made with unidirectional fiber sheet. Other properties include low-weight, high abrasion resistance, high electrical conductivity, low hygroscopicity, non-brittle failure, and resistance to biological rejection and chemical corrosion. Carbon-carbon composites are very workable, and can be formed into complex shapes.

Shortcomings

The chief drawback of carbon-carbon composites is that they oxidize readily at temperatures between 600-700°C, especially in the presence of atomic oxygen. A protective coating (usually silicon carbide) must be applied to prevent high-temperature oxidation, adding an additional manufacturing step and additional cost to the production process. The high electrical conductivity of airborne graphite particles creates an unhealthy environment for electrical equipment near machining areas. Carbon-carbon composites are currently very expensive and complicated to produce, which limits their use mostly to aerospace and defense applications.

Property	Fine Grained Graphite	Unidirectional Fibres	3-D Fibres

Elastic Modulus (GPa)	10-15	120-150	40-100
Tensile Strength (MPa)	40-60	600-700	200-350
Compressive Strength (MPa)	110-200	500-800	150-200
Fracture Energy (kJm^{-2})	0.07-0.09	1.4-2.0	5-10
Oxidation resistance	v.low	poor	better than graphite

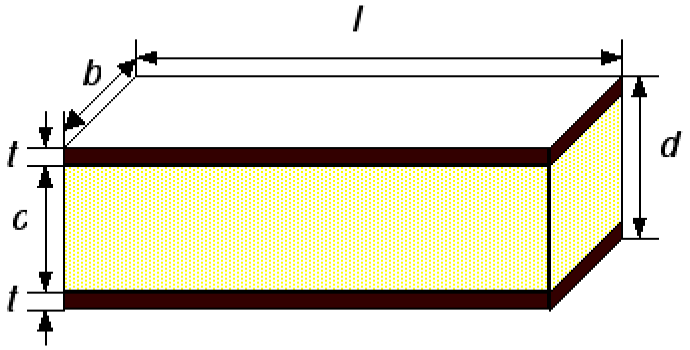
Properties of Carbon-Carbon Composites

Design and Properties of Sandwich Core Structures

Structural members made of two stiff, strong skins separated by a lightweight core are know as sandwich panels. The separation of the skins, which actually carry the load, by a low density core, increases the moment of inertia of the beam/panel with little increase in weight producing an efficient structure. Examples include skis, where a carbon fibre - epoxy skin is bonded to and separated by either a rigid polyurethane foam or by balsa wood, the fan blade of modern gas turbine jet engines where two titanium alloy skins (the front and rear surface of the blade are separated and bonded two a honycomb structure made from titanium sheet and the hulls of modern racing yachts where two thin layers of composite skin (glass/carbon/kevlar in vinyl ester/epoxy) are separated by a nomex core (nomex is a combination aluminium honeycomb filled with rigid polyurethane foam).

The mechanical properties of the sandwich depend on the core and face materials as well as the thickness of the core and faces. In most cases, the panels must have a minimum stiffness (deflection/unit load) and strength. The design process is essentially one of optimization wherein a property such as weight or stiffness/unit weight is minimized. A complete analysis can be found in “Cellular Solids - Structure and Properties” by Lorna Gibson & Mike Ashby.

Design for Stiffness - Weight Optimization



The simplest case to consider is that of point loading in 3-pt bend, though we shall show that the analysis is identical for other forms of bending load with the exception of the different values for the geometrical constants. The span of the beam is l, the width, b, the core thickness c, and the face thickness, t. The beam thickness is d where d = c+2t; we shall assume that t<<c and to a first approximation c~d. We shall denote the elastic modulus and density of the core as ρ_c^* and E_c^* where the * indicates a property of the foam rather than the material the foam is made from. Clearly the foam density and modulus are a function of the relative density of the core as well as the material the core is made from. To a first approximation these properties are given by

$$E_c^* = C_1 E_s \left(\frac{\rho_c^*}{\rho_s}\right)^2$$

$$G_c^* = C_2 E_s \left(\frac{\rho_c^*}{\rho_s}\right)^2$$

where E_s and ρ_s are properties of the bulk (100% solid) core material. C_1 (~1) and C_2 (~0.4) are constants We can also define ρ_f and E_f as the density and longitudinal stiffness of the face material.

The stiffness of thebeam in bending is calculated from the *equivalent flexural rigidity*, $(EI)_{eq}$, and the *equivalent shear rigidity*, $(AG)_{eq}$. Using the parallel axis theorem

$$(EI)_{eq} = \frac{E_f b t^3}{6} + \frac{E_c b c^3}{12} + \frac{E_f b t d^2}{2}$$

The first and second terms describe the stiffness of the two face sheets and the core while the third term adds the stifness of the faces about the centre of the beam. In a good beam design, the third term is substantially larger than the first two so if d~c then

$$(EI)_{eq} = \frac{E_f b t c^2}{2}$$

The equivalent shear rigidity is

$$(AG)_{eq} = \frac{G_c^* b d^2}{c} \approx b c G_c^*$$

When subject to a load, P, the deflection is the sum of the bending and shear components

$$\delta = \delta_b + \delta_s = \frac{P l^3}{B_1 (EI)_{eq}} + \frac{P l}{B_2 (AG)_{eq}}$$

where B_1 and B_2 are constants dependent on the geometry of the plate and the type of loading (Table 1). The compliance of the beam is

$$\frac{\delta}{P} = \frac{2l^3}{B_1 E_f b t c^2} + \frac{l}{B_2 b c G_c^*}$$

Table 1. Constants for bending and failure of beams

Mode of Loading	B_1	B_2	B_3	B_4
Cantilever, end load (P)	3	1	1	1
Cantilever, uniformly distributed load (P/l)	8	2	2	1
Three point Bend, central load (P)	48	4	4	2
Three point Bend, uniformly distributed load (P/l)	384/5	8	8	2
Ends built in, central load (P)	192	4	8	2
Ends built in, uniformly distributed load (P/l)	384	8	12	2

The key issue in design of most sandwich panels is the minimization of weight. The weight of the beam is

$$W = 2\rho_f g b l t + \rho_c^* g b l c$$

where g is the acceleration due to gravity. This is known as the “objective function” since this is what we wish to minimize. The span,l, the width, b, and the stiffness, P/□, are fixed by the design, the free variables are c,t and ρ_c^*. If the core density is fixed then the optimization is easy

1. Rewrite the stiffness equation in the form t=
2. Substitute the equation for t into the weight function
3. Differentiate the weight equation with respect to the other free variable, c and set equal to zero, find c_{opt}
4. Substitute c_{opt} into the “t” equation and find t_{opt}.
5. Using the objective function(the weight equation) determine the weight of the beam
6. Ask yourself... are c_{opt} and t_{opt} realistic/sensible ?
7. Modify c_{opt} and t_{opt} as necessary and reevaluate the weight.

.

The weight minimization problem may be better understood by employing a graphical solution.

Again, for simplicity, we shall assume that the core density os fixed. We now create a graph in which (t/l), thickness of the face /length of beam is plotted as a function of (c/l) core thickness to length (both of which are dimension less) and the two functions derived above, Weight and Stiffness are plotted. Before plotting, each equation must be rewritten such that (t/l) is a function of (c/l) ie.

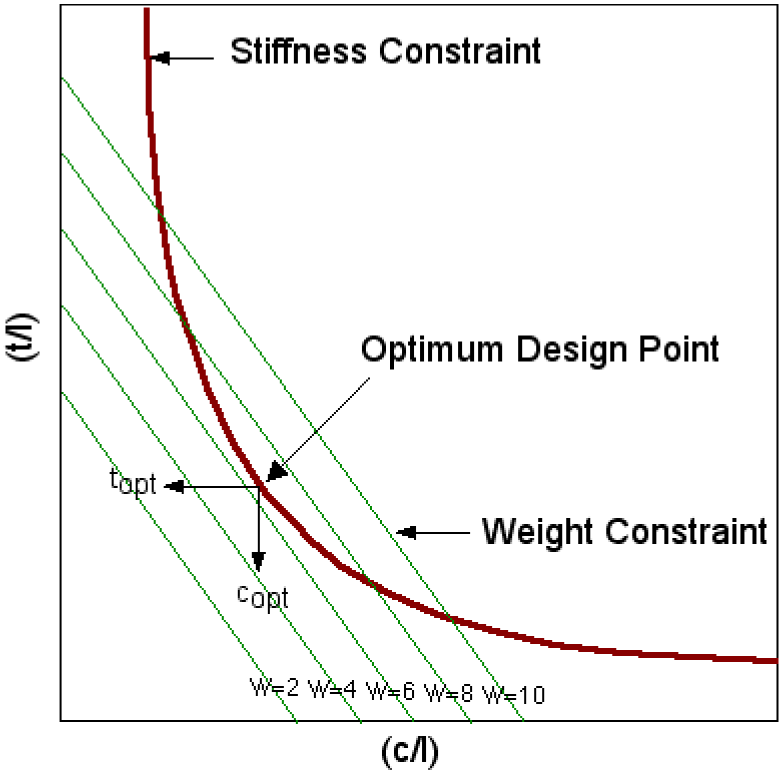
$$(t/l) = \frac{2B_2}{B_1} \cdot \frac{G_c^*}{E_f(c/l)} \cdot \left(\frac{1}{B_2(\delta/P)bG_c^*(c/l) - 1}\right)$$

the stiffness constraint which plots as curve, and

$$(t/l) = \frac{W}{2bl^2 \rho_f g} - \frac{\rho_c^*}{2\rho_f} \cdot (c/l)$$

which plots as a straight line for any given weight, or a series of parallel straight lines for different weights.

The two functions are plotted below, one (1) curve for a specific stiffness (P/□) and a series of parallel lines for successively increasing weights (W).



The optimum design point is found where the objective function (weight function) line is a tangent to the Stiffness Constraint. At this point the optimum (for minimum weight) values of face thickness, t, and core thickness (c) for a specified stiffness can be read off the graph.

If the core density were to be considered a free variable then a series of graphs are plotted for successively high core densities, the optimum values of c and t and hence weights determined then the actual minimum weight found. In reality, these ‘optimum’ values lead to too large core thickness' and too low core densities and some compromises are necessary for a realistic dsign.

Compression

The three main stages of deformation, when a stress is applied in-plane to a honeycomb are..

1. Elastic bending of cell walls
   i. Generalised Elastic behaviour of an Elastomer with temperature,
   ii. Generalised Elastic behaviour of an Polymer with temperature
   iii. Generalised Elastic behaviour of an Glassy/Brittle Polymer with temperature
2. Critical strain is attained and the cells start to collapse there are 3 possible collapbse processes that are determined by the material from which the cell walls are constructed.
   i. elastic buckling of the walls,
   ii. plastic yielding of walls,
   iii. brittle fracture

   During this stage the stress plateaus at the elastic buckling stress, the yield stress, or oscillates about a plateau as a series of brittle fracture events take place in successive cell walls.

3. Cell walls touch - densification, stress rises rapidly

The Stress-Strain plots are similar for all material types

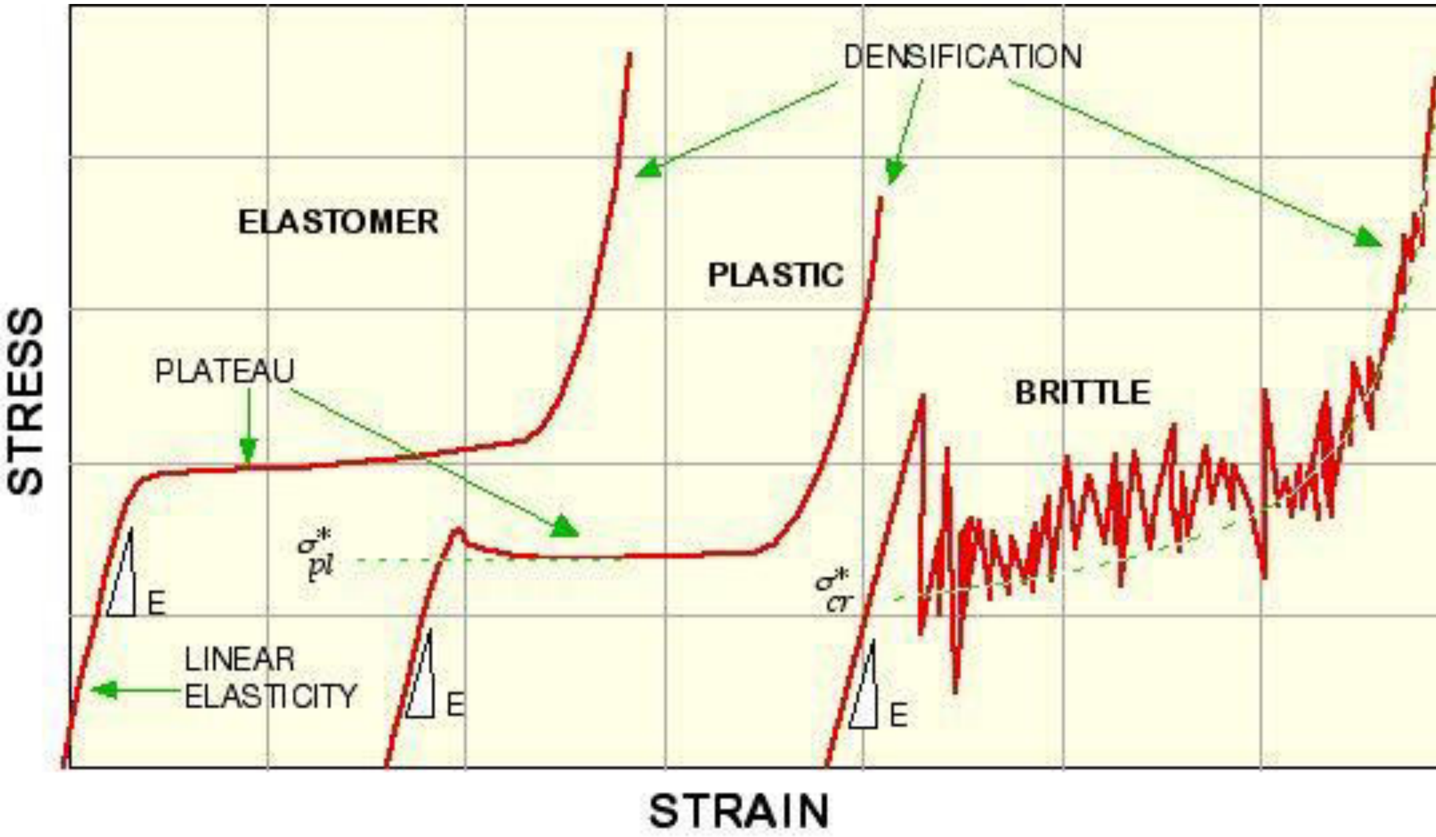


Fig.1 In-plane stress-strain curves for compression of elastomeric, elastic-plastic and elastic-brittle cell walls

Tension

The in-plane tensile behaviour of the honeycomb is simpler. There are two main stages..

1. Elastic bending of the cell walls - just in the opposite sense of the compression behaviour
2. In tension there is no buckling of the thin walls and so the second stage depends on the material
    - i. For elastomers, the cell walls continue to rotate into the tensile direction and the stiffness rises because of densification..
    - ii. plastic yielding of the hinges results in deformation at a constant stress in the material, but because the walls of the cell are approaching there is a gradual increase in density so the stress rises.
    - iii. hinges or walls will snap in a brittle fashion.

Any increase in density due to the number of cell walls per unit volume increasing (gradual densification) results in increase in elastic moduli, plastic yield stress and brittle fracture stress - just as in compression.

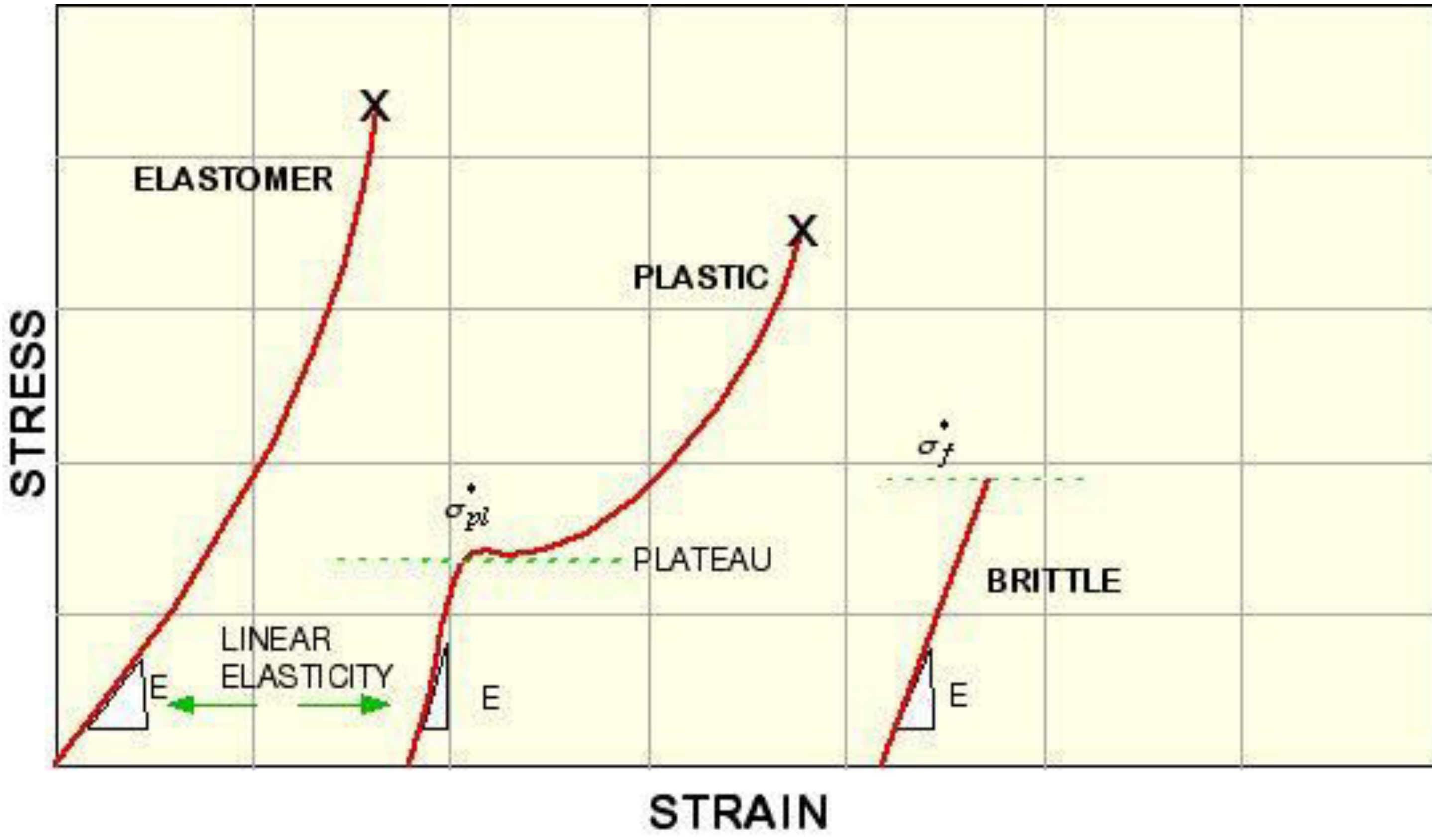
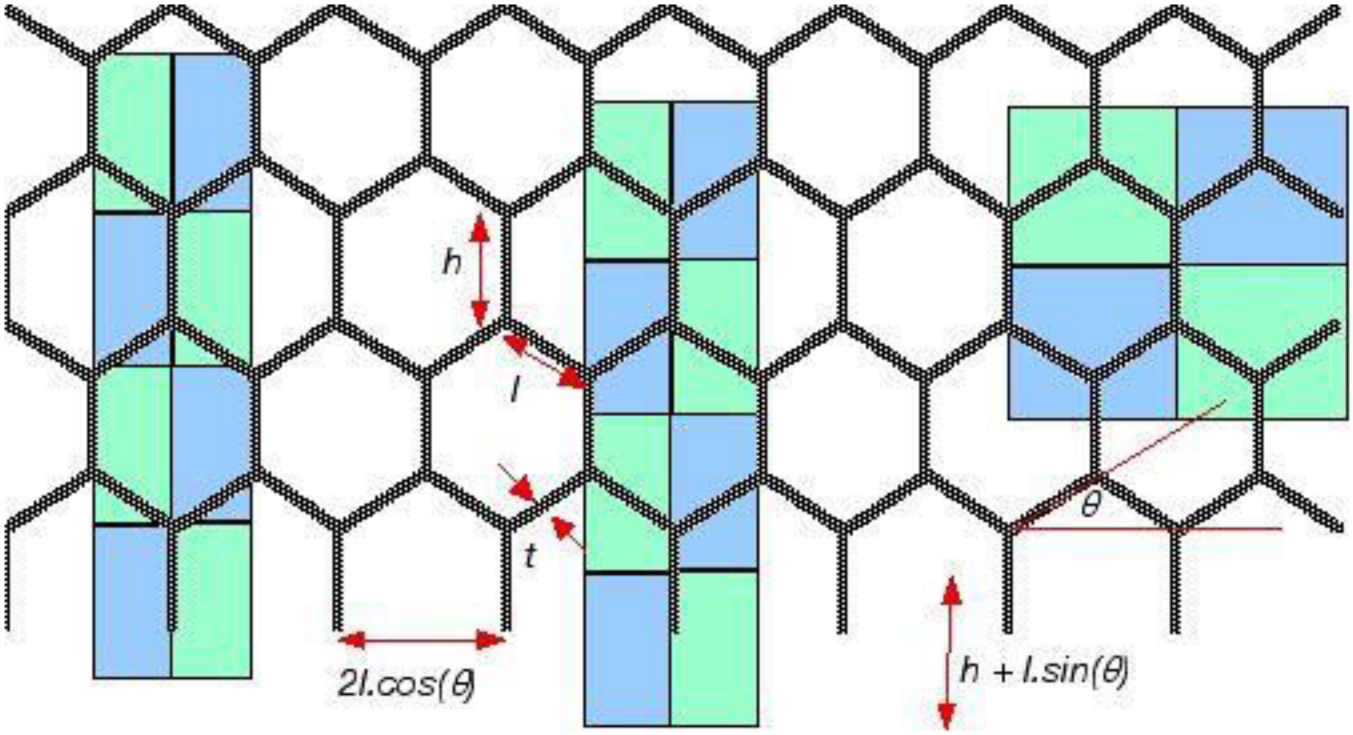


Fig.2 In-plane stress-strain curves for tension of elastomeric, elastic-plastic and elastic-brittle cell walls

Relative density of a generic honeycomb whose vertical sided have length h, whose angled sides have length l and the angle between h and l is 90+q, is calculated as follows;



Unit Cell Dimensions for Honeycombs

first determine the area of cell walls and area of cell.

$$A^{*} = \frac{(2h+4l)t}{2} = (h+2l)t$$

$$A = 2(l\sin\theta \cdot l\cos\theta) + h \cdot 2l\cos\theta = 2l\cos\theta(h + l\sin\theta)$$

should note that each cell wall is shared by two cells, hence the area of the walls is divided by 2.

$$\frac{\rho^{*}}{\rho_s} = \frac{t(2l+h)}{2l\cos\theta(h+l\sin\theta)}$$

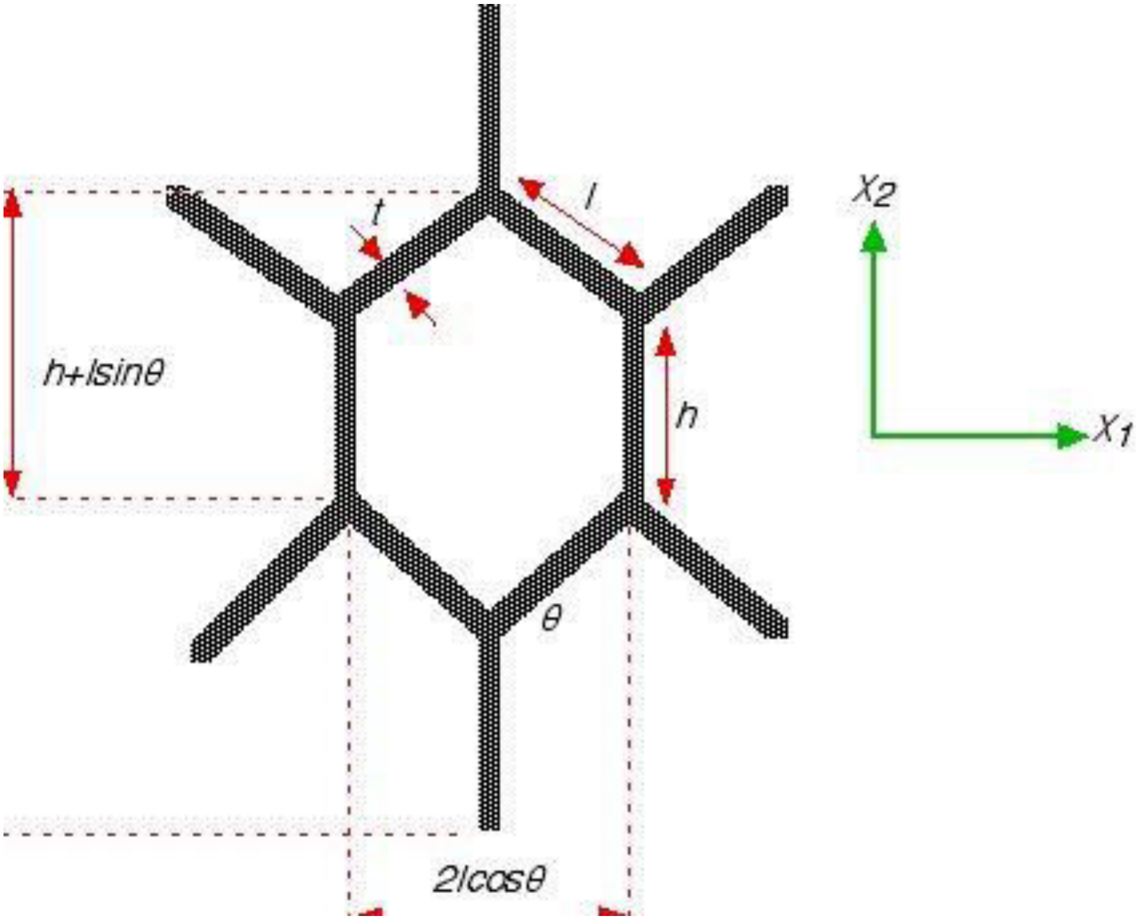
$$= \frac{t}{l} \cdot \frac{\left(\frac{h}{l}+2\right)}{2\cos\theta\left(\frac{h}{l}+\sin\theta\right)}$$

when the cells are regular hexagons, then q=30° and h=l, the relative density reduces to

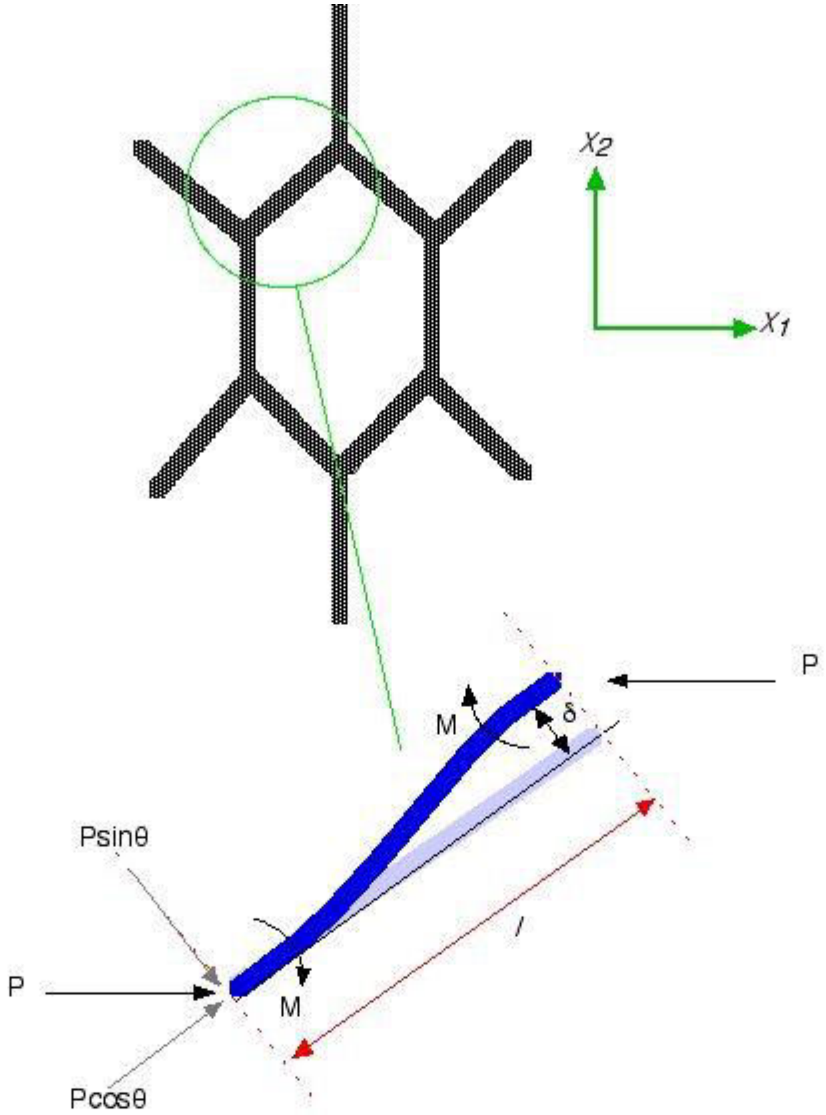
$$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l}$$

Linear Elasticity

There are 5 elastic constants for in-plane deformation, E_1,E_2,n_{12},n_{21} and G_{12}, however, only 4 are independent since $E_1\nu_{21} = E_2\nu_{12}$. E_1 and E_2 are determined as follows, using the geometry set out below where the remote stresses, s_1 and s_2 act in compression in directions X_1 and X_2. In the 1 direction the unit of area is $(h + l\sin\theta)b$ in the 2 direction the unit of area on which the stress acts is $(2l\cos\theta)b$ where b is the depth (out of plane) of the honeycomb.



deformation in the X_1 direction...



we can calculate the bending moment M, on the angled beam, from which we can determine the deflection, d,

$$M = \frac{P\sin\theta . l}{2} = \frac{\sigma_1(h + l\sin\theta)b\cdot l\sin\theta}{2}$$

$$\delta = \frac{Ml^2}{6EI} = \frac{\sigma_1(h + l\sin\theta)b\cdot l\sin\theta\cdot l^2}{12E_s\left(\frac{bt^3}{12}\right)}$$

The component of deflection in the X_1 direction is just $\delta\sin\theta$ acting over a length $l\cos\theta$ hence the strain in the X_1 direction is..

$$\varepsilon_1 = \frac{\delta \sin\theta}{l\cos\theta} = \frac{\sigma_1(h + l\sin\theta)b \cdot l\sin\theta \cdot l^2 \sin\theta}{12 E_s\left(\frac{bt^3}{12}\right) l\cos\theta}$$

$$= \frac{\sigma_1\left(\frac{h}{l} + \sin\theta\right) l^3 \sin^2\theta}{E_s t^3 \cos\theta}$$

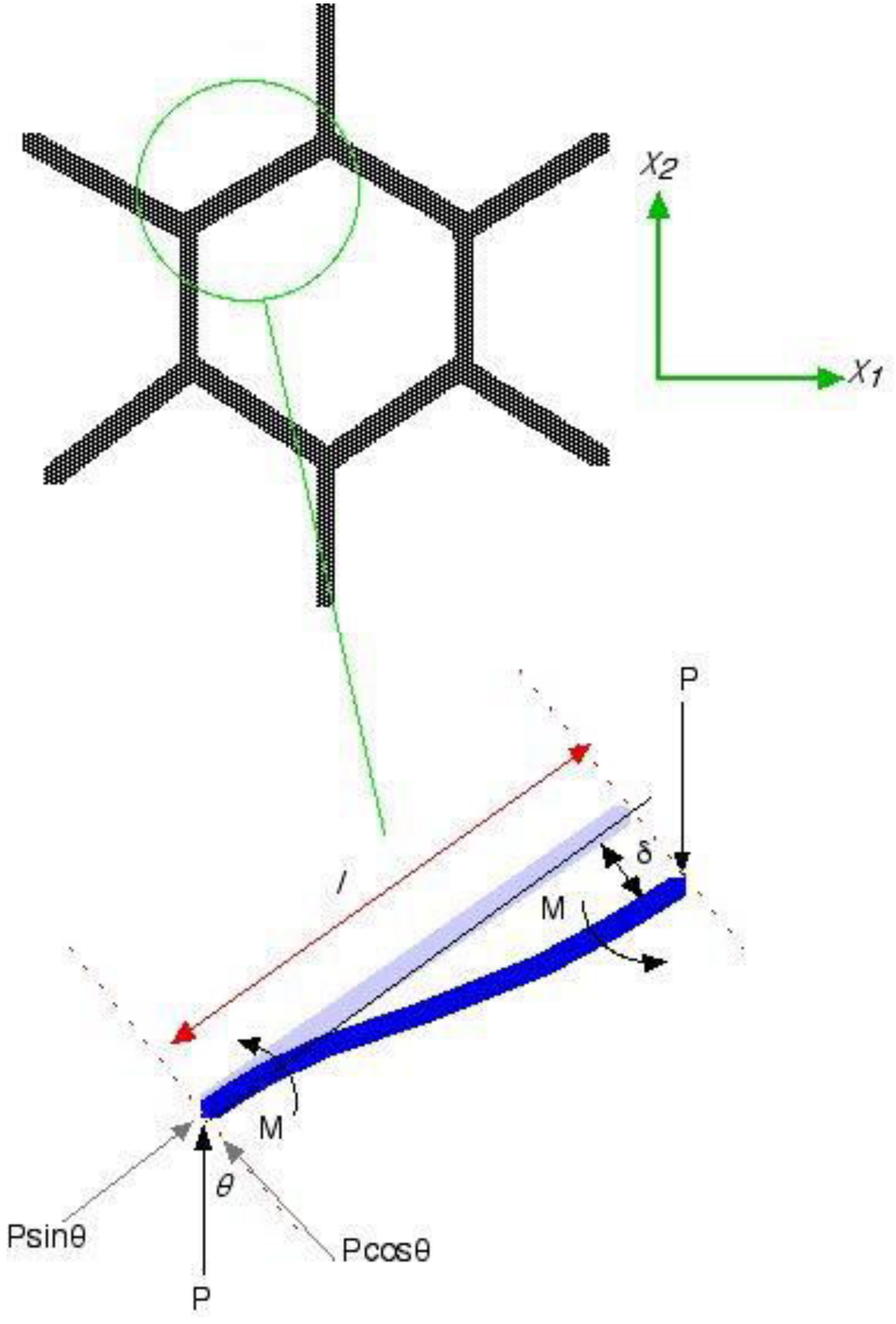
Since modulus is just the ratio of stress to strain, we can re-arrange the above equation to be

$$\frac{E_1^*}{E_s} = \left(\frac{t}{l}\right)^3 \frac{\cos\theta}{\left(\frac{h}{l} + \sin\theta\right)\sin^2\theta}$$

where E_s is the modulus of the solid of which the beam is made. When the honeycomb consists of regular hexagons in which h=l and q is 30° then the above equation reduces to

$$\frac{E_1^*}{E_s} = \frac{4}{\sqrt{3}}\left(\frac{t}{l}\right)^3$$

Now let’s repeat the exercise for a stress applied in the X_2 direction



The bending moment, M and deflection d are...

$$M = \frac{Pl\cos\theta}{2} = \frac{\sigma_2 l\cos\theta b \cdot l\cos\theta}{2}$$

$$\delta = \frac{Ml^2}{6EI} = \frac{\sigma_2 l\cos\theta b \cdot l\cos\theta \cdot l^2}{12 E_s\left(\frac{bt^3}{12}\right)}$$

The component of deflection in the X_2 direction is just $\delta\cos\theta$ acting over a length $h + l\cos\theta$ hence the strain in the X_2 direction is..

$$\varepsilon_2 = \frac{\delta\cos\theta}{h + l\sin\theta} = \frac{\sigma_2 l\cos\theta b \cdot l\cos\theta \cdot l^2\cos\theta}{12 E_s\left(\frac{bt^3}{12}\right)(h + l\sin\theta)}$$

$$= \frac{\sigma_2 l^3\cos^3\theta}{E_s t^3\left(\frac{h}{l} + \sin\theta\right)}$$

Since modulus is just the ratio of stress to strain, we can re-arrange the above equation to be

$$\frac{E_2^*}{E_s} = \left(\frac{t}{l}\right)^3 \frac{\left(\frac{h}{l} + \sin\theta\right)}{\cos^3\theta}$$

where E_s is the modulus of the solid of which the beam is made. When the honeycomb consists of regular hexagons in which h=l and q is 30° then the above equation reduces to

$$\frac{E_2^*}{E_s} = \frac{4}{\sqrt{3}}\left(\frac{t}{l}\right)^3$$

which is the same as for E_1 and so a regular hexagonal honeycomb is isotropic: they are in practice!

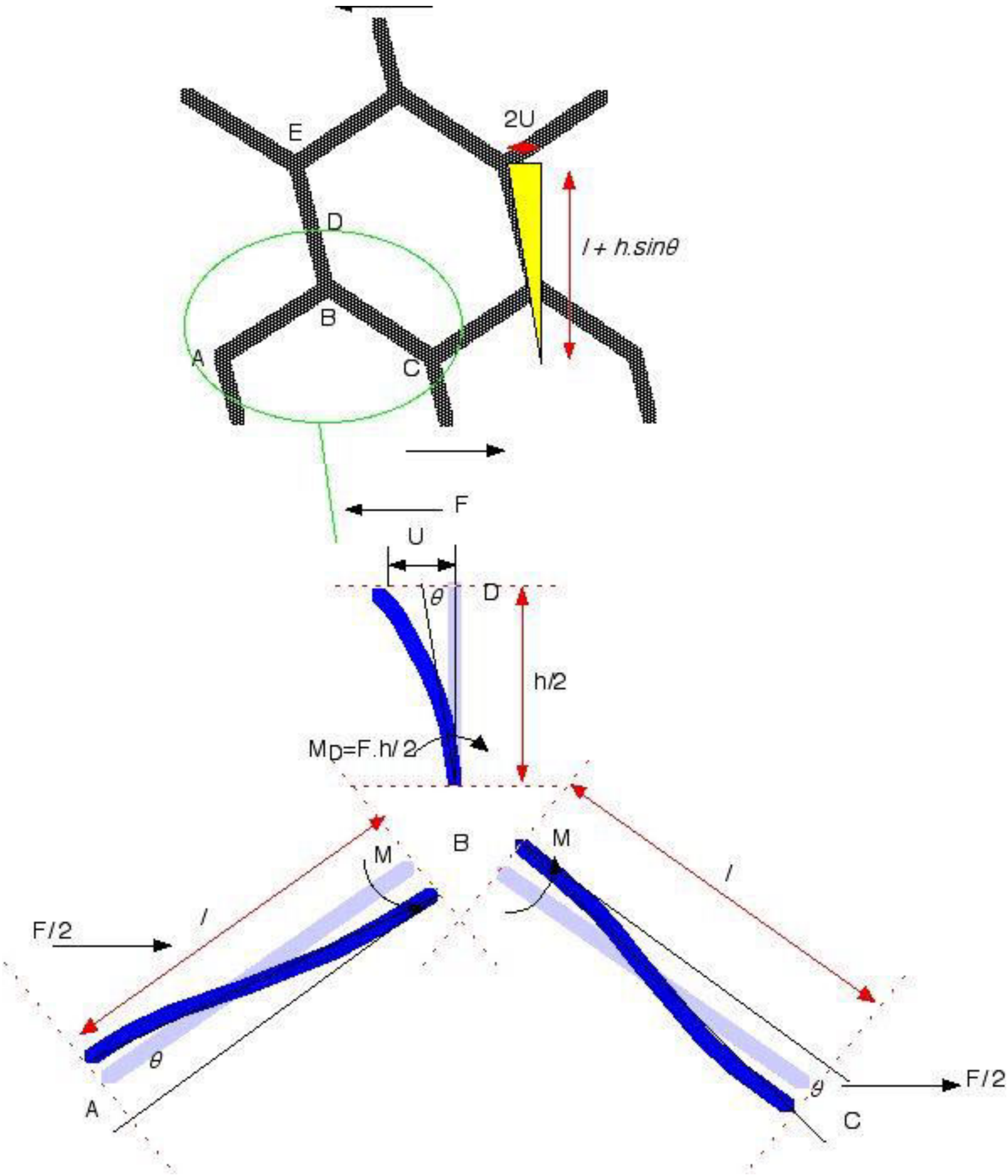
We can now evaluate poisson's ratio by taking the negative strain in the normal direction and dividing by the strain in the loading direction..

$$\nu_{12}^* = -\frac{\varepsilon_2}{\varepsilon_1} = \frac{\delta\cos\theta / (h + l\sin\theta)}{\delta\sin\theta / l\cos\theta} = \frac{\cos^2\theta}{\left(\frac{h}{l} + \sin\theta\right)\sin\theta}$$

which for a regular hexagon is equal to 1, thus $\nu_{12}^* = \nu_{21}^* = 1$. For non-regular hexagons

$$\nu_{21}^* = -\frac{\varepsilon_1}{\varepsilon_2} = \frac{\delta\sin\theta / l\cos\theta}{\delta\cos\theta / (h + l\sin\theta)} = \frac{\left(\frac{h}{l} + \sin\theta\right)\sin\theta}{\cos^2\theta}$$

Finally, we need to establish the inplane shear modulus G_{12}... In the drawing below we apply a shear to th ehoneycomb and assume that the shear actually takes place by the vertical wall buckling over, while the apeces A,B and C remain fixed relative to each other with only the point D moving laterally.



The bending moment at the bottom of the arm DB is simply the product of force F and distance h/2. This is balanced by the sum of the bending moments in arms AB and BC so $M = \frac{Fh}{4}$. The deflection in the arms AB and BC is then $\delta = \frac{Ml^2}{6E_sI}$ and the angle through which the beam rotates is $\theta = \frac{Ml}{6E_sI}$. The deflection of point D in the horizontal plane is made up of the deflection due to the rotation of the hinge through an angle q, because of the rotation in AB and BC which must be equal to each other as well as that in the arm BD; plus the bending of an end loaded cantilever beam BD due to the force F hence

$$U = \frac{h}{2}\tan\theta + \frac{F}{3E_sI}\left(\frac{h}{2}\right)^3$$

If q is small the q =tanq so..

$$U = \frac{h}{2}\cdot\frac{Fhl}{24E_sI} + \frac{F}{3E_sI}\frac{h^3}{8}$$

$$= \frac{Fh^2}{48E_sI}(l+2h)$$

The opposite end of the arm BD, point E moves a horizontal distance twice that of the point D and so the shear strain is $\gamma = \frac{2U}{h + l\sin\theta}$, while the remote stress is $\tau = \frac{F}{2l\cos\theta\cdot b}$, giving the shear modulus as

$$G_{12}^{\bullet} = \frac{\tau}{\gamma} = \frac{\left(\frac{F}{2l\cos\theta\cdot b}\right)}{\left(\frac{2U}{h + l\sin\theta}\right)} = \frac{F}{2l\cos\theta\cdot b}\frac{h + l\sin\theta}{2U}$$

$$= \frac{F(h + l\sin\theta)}{4lb\cos\theta}\frac{48E_s\left(\frac{bt^3}{12}\right)}{Fh^2(l+2h)}$$

$$\frac{G_{12}^{\bullet}}{E_s} = \left(\frac{t}{l}\right)^3\frac{\left(\frac{h}{l} + \sin\theta\right)}{\left(\frac{h}{l}\right)^2\left(1 + \frac{2h}{l}\right)\cos\theta}$$

when the cells are regular hexagons then

$$\frac{G_{12}^{\bullet}}{E_s} = \frac{1}{\sqrt{3}}\left(\frac{t}{l}\right)^3 = \frac{1}{4}\frac{E^{\bullet}}{E_s}$$

which obeys the standard relation for isotropic solids $G = E/2(1+\nu)$recalling that v=1

...

Mechanics of Honeycombs - Plateau Stress

Now that we have established approximations for the in-plane elastic properties of the honeycombe we next ne determine the yield, fracture or plateau stress for the honeycomb. The actual behaviour of the honeycomb in th stage of deformation will depend on the type of material from which the honeycomb is made.

Elastomeric Materials

The plateau of the stress strain curve of a honeycomb made from an elastomeric material in compression in the direction is associated with the onset of elastic buckling of the thin walls that are aligned nearly parallel to the a stress - and is simply the point at which the load acting on the end of the column exceeds the Euler buckling lo

$$P_{crit} = \frac{n^2 \pi^2 E_s I}{h^2}$$

The factor n describes the rotational stiffness of the end of the column where the 3 cell walls meet. n can vary f in a system which is completely free to rotate to 2 in one in whch the ends are held rigid. For regular hexagons value is close to 0.69. The load on the end of each column is simply

$$P = 2\sigma_2 lb\cos\theta$$

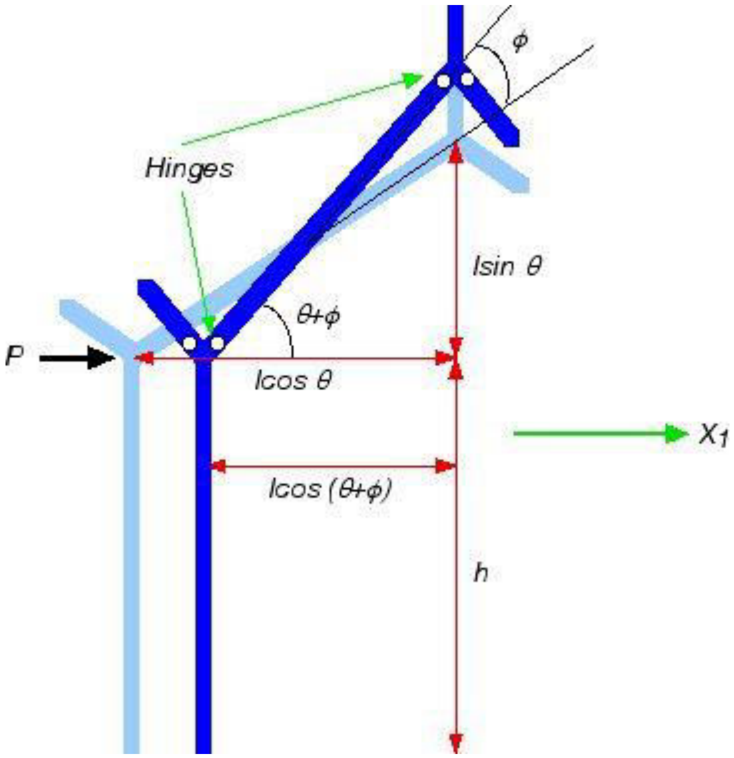
so the elastic collapse stress in the X_2 direction is simply...

$$\frac{\overset{\bullet}{\sigma}_{el_2}}{E_s} = \frac{n^2\pi^2}{24}\frac{t^3}{lh^2\cos\theta} \text{ for regular hexagons } \frac{\overset{\bullet}{\sigma}_{el_2}}{E_s} = 0.22\left(\frac{t}{l}\right)^3$$

In the X_1 direction the walls continue to bend. In tension, buckling cannot occur and the cell walls just continue andthe cell volume decreases - this leads to a rise in apparent density and hence the stress continues to climb the number of cell walls per unit area perpendicular to the applied stress increases with increasing strain.

Metals and Plastic Polymers

In an elastic plastic solid the bending moment on the cell walls will eventually reach the point where it becomes plastic and the cell wall “gains” a plastic hinge which will slowly fold over with iincreasing strain at an almost co stress - polymers, or slowly rising stress if there is significant work hardening - metals. On way to determine the at which the cell wall plastic hinge develops is to equate the work done by the external stress moving the cell w inwards with the work done actually bending the cell wall..



The force P, acting on the end of the cell wall in the X_1 direction is simply

$$P = \sigma_1(h + l\sin\theta)b$$

The distance moved in the X_2 direction on each side of the cell is

$$\begin{aligned} X &= l\cos\theta - l\cos(\theta+\phi) \\ &= l(\cos\theta - (\cos\theta\cos\phi - \sin\theta\sin\phi)) \\ &= l\phi\sin\theta \end{aligned}$$

if f is small (cos(f)=1 and sin(f) is f). ***Is this a valid approximation?***

The work done in each of the 4 plastic hinges is simly the product of the moment (M) and the angular rotation (f the work done bythe external force is the product of force (P) and the total distance moved (2X; X on each side cell) so..

$$4M\phi = 2\sigma_1(h + l\sin\theta)b \cdot l\phi\sin\theta$$

The plastic moment for a rectanglar section beam is given by

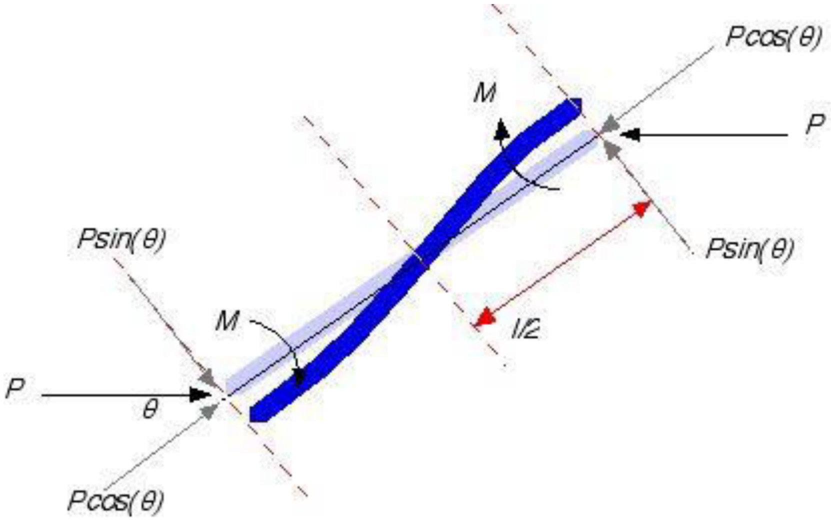
$$M = H\sigma_{ys} = \frac{bt^2}{4}\sigma_{ys}$$

re-arranging we find..

$$\frac{\sigma^*_{pl_1}}{\sigma_{ys}} = 4\phi\frac{bt^2}{4}\cdot\frac{1}{2(h + l\sin\theta)b\cdot l\phi\sin\theta}$$

$$\frac{\sigma^*_{pl_1}}{\sigma_{ys}} = \left(\frac{t}{l}\right)^2\frac{1}{2\left(\frac{h}{l}+\sin\theta\right)\sin\theta}$$

There is a second way to find the plastic collapse stress and that is by setting the maximum bending moment in wall to the moment required for full plasticity in the cross-section of the beam. The geometry is..



The bending moment is

$$M = P\sin\theta\cdot\frac{l}{2} = \sigma_1(h + l\sin\theta)b\cdot\sin\theta\cdot\frac{l}{2}$$

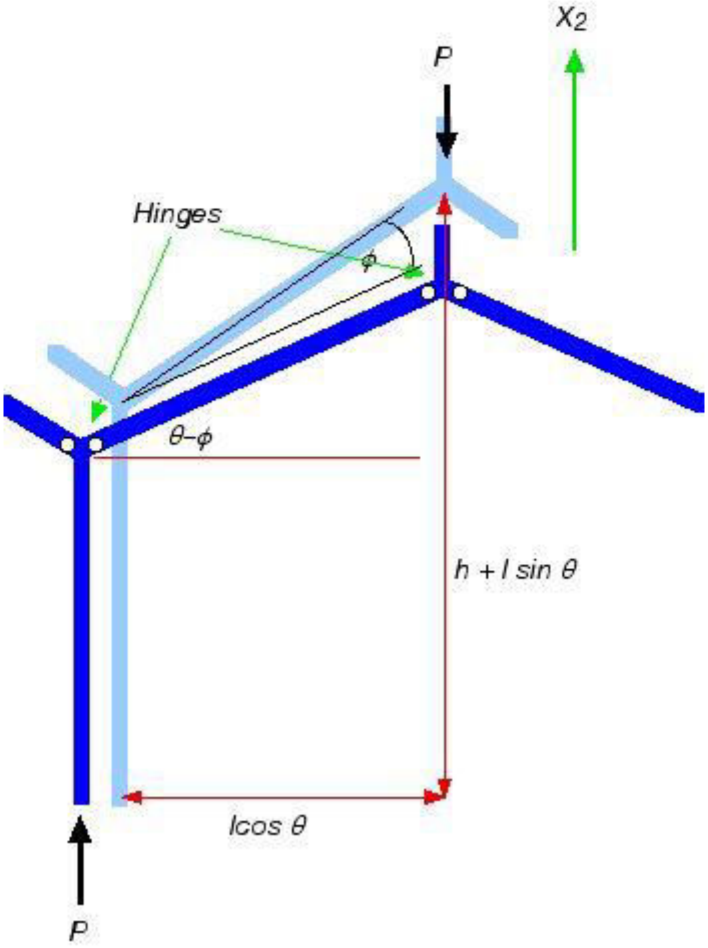
while the moment for full plasticity is

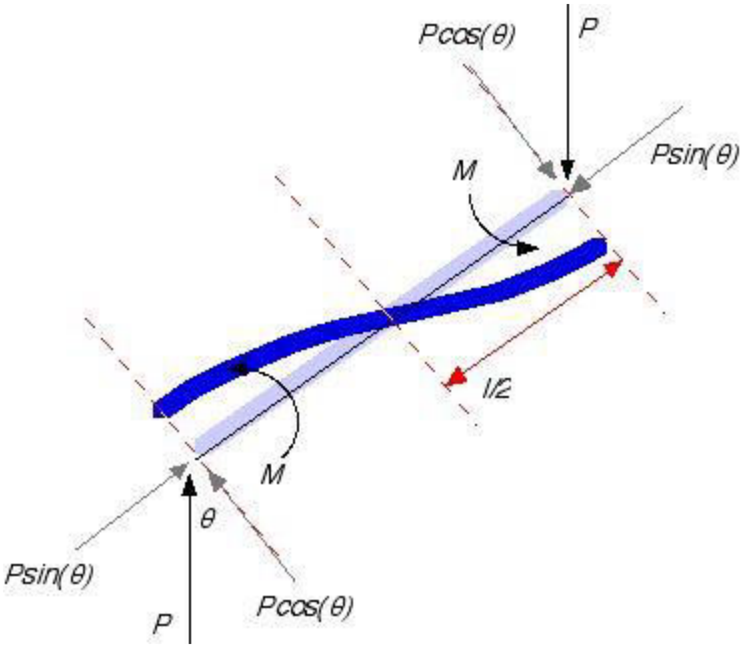
$$M = \frac{bt^2}{4}\sigma_{ys}$$

Combining the two equations leads to exactly the same aanswer as before. For regular hexagons the stress for onset of plasticity reduces to

$$\frac{\sigma^*_{pl_1}}{\sigma_{ys}} = \frac{2}{3}\left(\frac{t}{l}\right)^2$$

Since both approaches (work doen and maximum bending moment produce the same result - we shall just use latter method for determining the onset of plasticity in the hinges when deforming in the X_2 direction.





The bending moment is

$$M = P\cos\theta \cdot \frac{l}{2} = \sigma_2\left(l\cos\theta\right)b \cdot \cos\theta \cdot \frac{l}{2}$$

while the moment for full plasticity is

$$M = \frac{bt^2}{4}\sigma_{ys}$$

Combining the two equations gives..

$$\frac{\overset{\bullet}{\sigma}_{pl_2}}{\sigma_{ys}} = \left(\frac{t}{l}\right)^2 \frac{1}{2\cos^2\theta}$$

For regular hexagons the stress for the onset of plasticity reduces to

$$\frac{\overset{\bullet}{\sigma}_{pl_2}}{\sigma_{ys}} = \frac{2}{3}\left(\frac{t}{l}\right)^2$$

which is exactly the same as for the X_1 direction and so the stress for the onset of plasticity in the walls of the ce
isotropic.

Plastic Collapse in Shear with the honeycomb loaded in the X_1-X_2 plane then plastic hinges form in the vertica
The shear force acting on the walls F , creates a moment M, which must exceed the fully plastic moment of the

$$F = 2\tau_{12} lb\cos\theta \quad M = F\frac{h}{2} = \tau_{12} lbh\cos\theta = \sigma_{ys}\frac{bt^2}{4}$$

re-arranging

$$\frac{\tau_{12}}{\sigma_{ys}} = \frac{1}{4}\left(\frac{t}{l}\right)^2 \frac{1}{\frac{h}{l}\cos\theta}$$

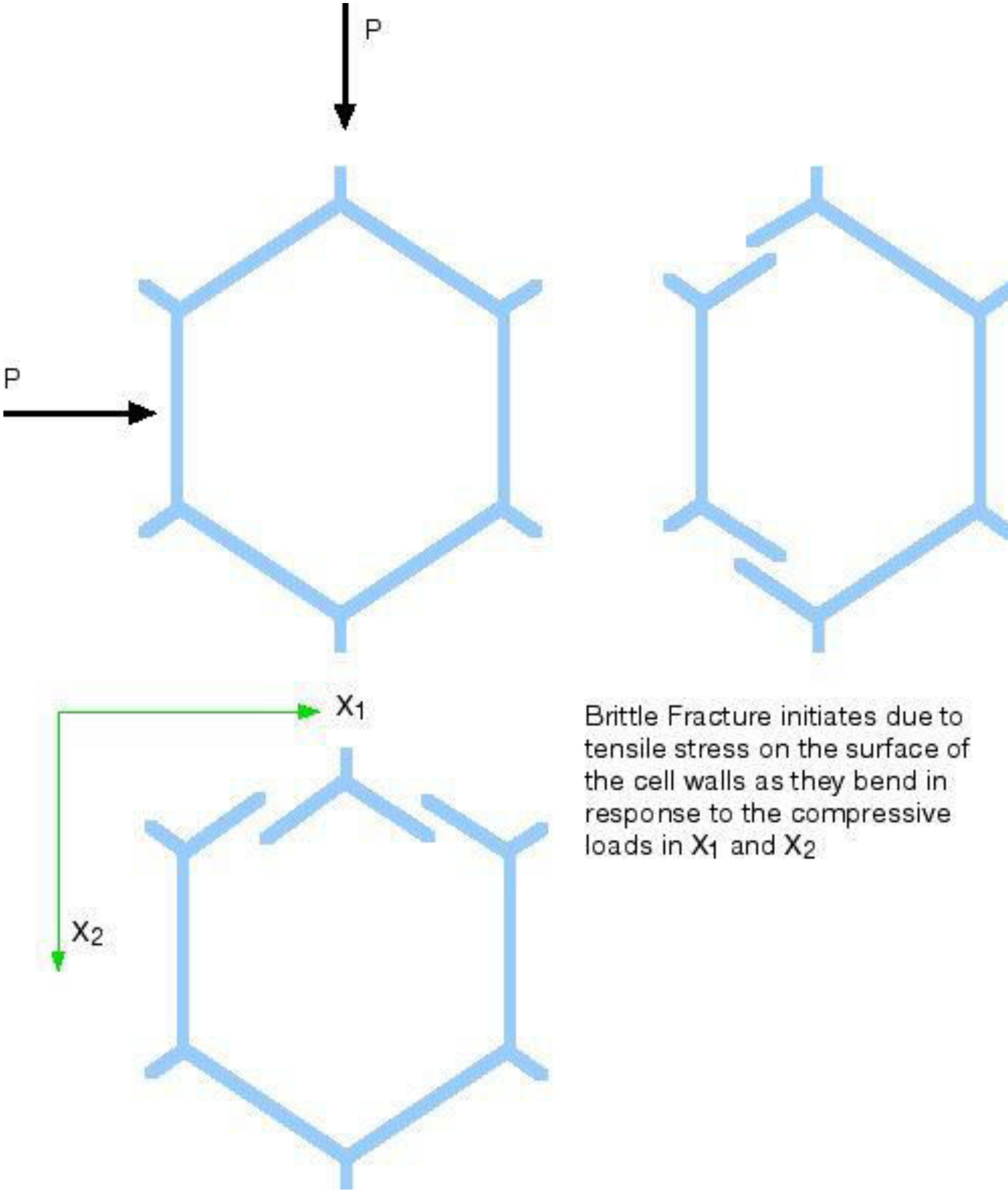
which for regular hexagons reduces to

$$\frac{\tau_{12}}{\sigma_{ys}} = \frac{1}{2\sqrt{3}}\left(\frac{t}{l}\right)^2$$

Mechanics of Honeycombs - Brittle Fracture

Brittle Failure

Brittle failure can occur in both tension and compression if the cell wall material itself is brittle; materials such as glasses, ceramics, brittle plastics like polystyrene, suffer progressive crushing in compression and fast crack gr tension. We have already demonstrated that in compression the cell walls bend and buckle, leading to the deve of tensile stresses on some portions of the outer surfaces of the cell walls - it is these regions of tension that init crack propagation through the cell wall and fracture.



Brittle Fracture initiates due to tensile stress on the surface of the cell walls as they bend in response to the compressive loads in X_1 and X_2

The maximum surface stress on the cell wall is simply

$$\sigma_{\max} = \frac{6M}{bt^2}$$

if we ignore the axial compression of the cell wall by the component of the applied stress resolved parallel to the wall. The moment was calculated in the previous sections and depends on the direction of loading..

$$M_1 = \frac{1}{2}\sigma_1(h + l\sin\theta)bl\sin\theta \quad M_2 = \frac{1}{2}\sigma_2 bl^2\cos^2\theta$$

hence, if $\sigma_{\max} = \sigma_{fs}$ the fracture strength of the cell wall material, then

$$\frac{\sigma_{cr_1}}{\sigma_{fs}} = \left(\frac{t}{l}\right)^2 \frac{1}{3\left(\frac{h}{l} + \sin\theta\right)\sin\theta} \quad \frac{\sigma_{cr_2}}{\sigma_{fs}} = \left(\frac{t}{l}\right)^2 \frac{1}{3\cos^2\theta}$$

which for regular hexagons both reduce to

$$\frac{\sigma_{cr}}{\sigma_{fs}} = \frac{4}{9}\left(\frac{t}{l}\right)^2$$

so the crushing strength is isotropic and scales with the square of the relative density, as do all the modes ofpla deformation.

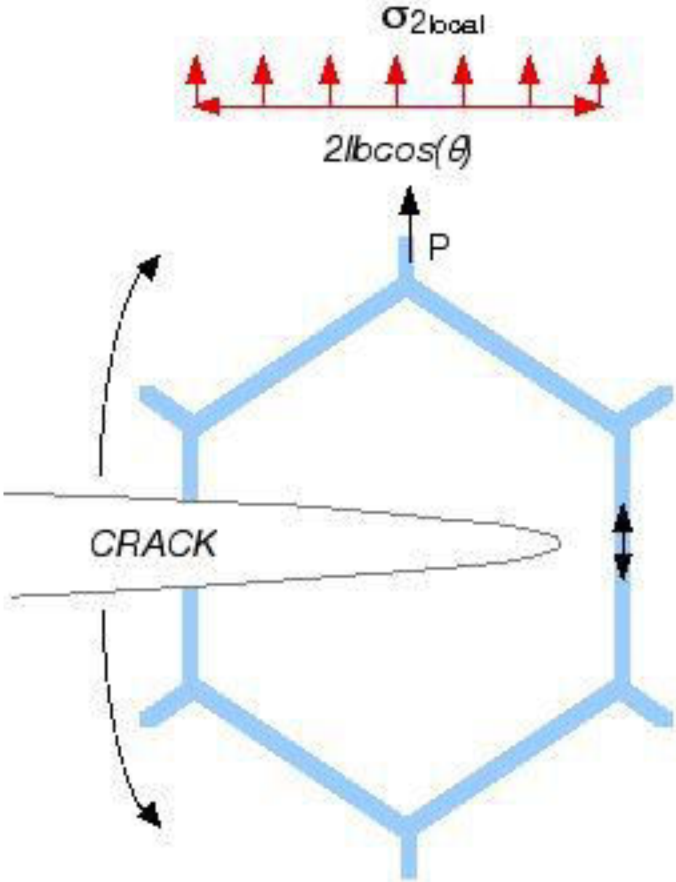
Tensile Failure - Fast Fracture

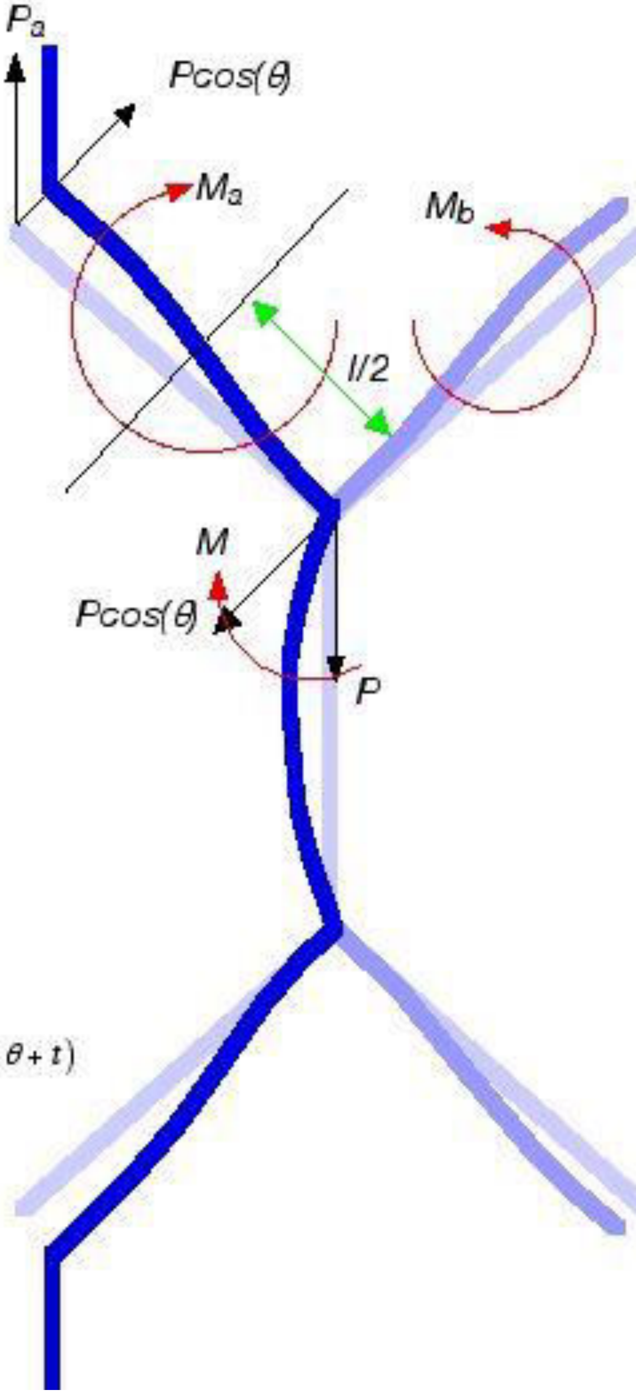
In tension the failure process is different. At some point a cell wall will fail in tension, adjacent walls will also fail load transfers to them and create what is effectively a macroscopic crack - an example is shown below. In a full body, the stress field ahead of the crack is defined by

$$\sigma_{local} = \frac{\sigma^{\infty}\sqrt{\pi c}}{\sqrt{2\pi r}}$$

where r is the distance ahead of the crack tip and σ^{∞} is the remote stress perpendicular to the plane of the crac opening of the crack creates a bending moment on the cell wall immediately ahead of the crack tip .

The bending moment in the vertical beam ahead of the crack exists because of the gradient in stress ahead of t tip which results in a larger force pulling on the first column above the end of the crack, than in the second and t columns, ie. the moments in the two angled beams do not cancel out leaving a resdual moment rotating the ver beam ahead of the crack tip. This moment creates a tensile stress on the crack side of the vertical cell wall. The bending stress is supplemented by the actual tensile stress in the cell wall itself.





$$M = M_a - M_b \propto P\cos\theta \cdot \frac{l}{2}$$

$$\sigma = M\frac{y}{I} + \frac{P}{A} = \frac{Pl\cos\theta}{2}\frac{t/2}{bt^3/12} + \frac{P}{bt} = \frac{P}{bt^2}(3l\cos\theta + t)$$

since we shall assume that $M_b << M_a$ and $l >> t$ then we can ignore the second term and fracture will commence w stress reaches the failure stess of the cell wall material. On average $P = \sigma_{2_{local}} 2lb\cos\theta$, where the the stress i the crack tip hence taking r, the distance ahead of the crack tip as half the width of the cell then...

$$\sigma_{fs} = 2\sigma_{2_{local}} lb\cos\theta \cdot 3l\cos\theta \cdot \frac{1}{bt^2}$$

$$= 2\sigma_2 \sqrt{\frac{c}{2l\cos\theta}} \cdot lb\cos\theta \cdot 3l\cos\theta \cdot \frac{1}{bt^2}$$

therefore

$$\frac{\sigma^{\bullet}_{fs_2}}{\sigma_{fs}} = \frac{1}{3\sqrt{2\cos^3\theta}}\sqrt{\frac{l}{c}}\left(\frac{t}{l}\right)^2$$

remembering that this is an approximation. The same approach can be used for crack propagation when stress
X_1 direction.

$$\sigma = M\frac{y}{I} + \frac{F}{A} = Pl\sin\theta \frac{t/2}{bt^3/12} + \frac{P\cos\theta}{bt} = \frac{P}{bt^2}(6l\sin\theta + t\cos\theta)$$

On average, taking the crack to occupy hallf the width of the unit cell, then

$$P = \sigma_{1_{local}}(h + l\sin\theta)b$$

$$P = \sigma_1 \frac{\sqrt{\pi c}}{\sqrt{2\pi(h + l\sin\theta)/2}}(h + l\sin\theta)b$$

If fracture occurs when the stress exceds the fracture strength of the cell wall material, and l>>t then

$$\frac{\sigma^{\bullet}_{fs_1}}{\sigma_{fs}} = \frac{1}{6\sqrt{\left(\frac{h}{l} + \sin\theta\right)}}\sqrt{\frac{l}{c}}\left(\frac{t}{l}\right)^2$$

For a regular haxagonal cell structure, both equations for the fracture strength in tension reduce to the same va
the fracture strength is isotropic

$$\frac{\sigma^{\bullet}_{f}}{\sigma_{fs}} = 0.3\sqrt{\frac{l}{c}}\left(\frac{t}{l}\right)^2$$

If we re-write the above equation in terms of fracture toughness, where $K = \sigma\sqrt{\pi c}$ then

$$K^{\bullet}_{Ic} = 0.3\sigma_{fs}\sqrt{\pi l} \cdot \left(\frac{t}{l}\right)^2$$

Properties of Selected Fibres

	Density (gcm^{-3})	Longitudinal Tensile Modulus	Transverse Tensile Modulus	Poisson's ratio	Shear Mudulus G_{12}	Longitudinal Tensile Strength	Longitudinal Thermal Expansion	Transverse Thermal Expansion	Heat Resistan

		E_1 (GPa)	E_2 (GPa)	$□_{12}$	(GPa)	□ (MPa)	$□_1$ (10^{-6} K^{-1})	$□_2$ (10^{-6} K^{-1})	°C
Glass	2.45	71	71	0.22	30	3500	5	5	
PBT	1.58	365				5800			
Kevlar (49) Kevlar (29)	1.47	154 61	4.2	0.35	2.9	2800	-4	54	550 450
PE (Spectra) PE (Dyneema)	0.97 0.975	66-124[1] 115				2300-3250[2] 3500	- -12		150
PBO Zylon AS PBO Zylon HM	1.54 1.56	180 270				5800	-6		650
Graphite (AS)	1.75	224	14	0.2	14	2100	-1	10	
Graphite (HMS)	1.94	385	6.3	0.2	7.7	1750	-1	10	
Boron	2.45	420	420	0.2	170	3500	5	5	
SiC	3.2	406	406	0.2	169	3395	5.2	5.2	
Saffil (5%SiO_2-Al_2O_3)	3.3	300	300	0.2	126	1500	5.2	5.2	
Al_2O_3	3.9	385	385	0.3	154	1400	8.5	8.5	

[1] Spectra 900 E=66-73GPa; Spectra 1000 E=98-113GPa; Spectra 2000 E=113-124GPa

[2] Spectra 900 □=2.1-2.6GPa; Spectra 1000 □=2.9-3.25GPa; Spectra 2000 □=2.9-3.5GPa

PBT = Poly(p-phenylene-2,6-benzobisthiazole)

PE = Gel Spun ultra high molecular weight polyEthylene (Spectra®,Dyneema®)

PEN = polyEthylene Napthalate (Pentex®

PBO = Poly(p-phenylene-2,6-benzobisoxazole)

Properties of Seleced Matrices

	Density (gcm^{-3})	Tensile Modulus E	Poisson's ratio	Shear Mudulus G	Longitudinal Tensile Strength	Longitudinal Thermal Expansion	Glass transition melting

		(GPa)	□	(GPa)	□ (MPa)	□ (10^{-6} K^{-1})	Point (°C)
Epoxy	1.54	3.5	0.33	1.25	60	57.5	
PolyEster	1.38	2.5	0.33	1.2	35		260
polyVinylEster	1.5	3.0			60		260 (T_{max})
PolySulfone (pS)	1.25	2.7			106	56	190 (T_g)
PolyPhenyleneSulfone (pPS)	1.25	4.8			135	56	93 (T_g) 285 (T_m)
PolyEtherSulfone (pES)	1.37	2.6			129	55	230 (T_g)
PolyEtherImide (pEI)	1.27	3.3			145	62	217 (T_g)
PolyAmideImide (pAI)	1.4	4.8			197	63	
polyAmide (nylon)	1.36	2.5			100		
PEEK	1.3	3.6	0.37	1.4	70	45	143 (T_g) 334 (T_m)
Aluminium	2.71	69	0.32	26	74*	23.6	660
Titanium	4.51	113.8	0.33		238**	8.4	1670
Magnesium	1.74	45.5	0.33	7.5	189	26	650
Borosilicate glass	2.23	63.7	0.21	28	90	3.25	

* Aluminium Alloys such as 2024, 6061 and 7075 can be used as a matrix and can be heat treated to give tensi strengths up to 550 MPa.
** Titanium alloys such as IMI 550, Ti-6Al-4V can be used as matrices and have yield strengths in the range 80 1500 MPa.

For shear modulus a good approximation is 3/8 of Elastic modulus.

Composite Materals Design - Effect of Orientation on Stiffness and Strength

In this section I'll show you how to use MathCAD to calculate the elastic constants of a simple fibre reinforced composite at any angle to the fibres. We will then use the functions developed to calculate the stresses and stra result in the composite when a given set of deflections or loads are applied to that composite material. Finally, v develop a function that will predict the uniaxial strength of a composite in which all the fibres are aligned paralle other for any fibre orientation relative to the applied stress.

As in all the previous examples we will need to define the mechanical properties of the matrix and fibre material remembering that the fibre may be anisotropic.

	1. An Isotropic Matrix.	**2. Isotropic/Anisotropic Fibres**	
The elastic properties	$E_m := 3.5 \cdot GPa$	$E_f := 72 \cdot GPa$	$E_{ft} := 72 \cdot GPa$
	$\nu_m := 0.33$	$\nu_f := 0.33$	
	$G_m := \dfrac{E_m}{2 \cdot (1 + \nu_m)}$	$G_f := \dfrac{E_f}{2(1 + \nu_f)}$	
The yield strength	$\sigma_m := 60 \cdot MPa$ $\quad \varepsilon_m := \dfrac{\sigma_m}{E_m}$	$\sigma_f := 2100 \cdot MPa$	$\varepsilon_f := \dfrac{\sigma_f}{E_f}$
The density	$\rho_m := 1.54 \cdot \dfrac{gm}{cm^3}$	$\rho_f := 2.54 \cdot \dfrac{gm}{cm^3}$	

Next the micromechanical equations that define the elastic properties of the aligned fibre composite are defined of the fraction of fibres f, since ultimately, we will shall want to calculate the properties of the material for any vo fraction of fibres.

$$E_x(f) := E_f \cdot f + E_m \cdot (1 - f) \qquad \nu_{12}(f) := f \cdot \nu_f + (1 - f) \cdot \nu_m$$

$$E_y(f) := \frac{E_m \cdot E_{ft}}{f \cdot E_m + (1 - f) \cdot E_{ft}} \qquad \nu_{21}(f) := \upsilon_{12}(f) \cdot \frac{E_y(f)}{E_x(f)}$$

$$G(f) := \frac{G_m \cdot G_f}{f \cdot G_m + (1 - f) \cdot G_f}$$

$$G(f) := G_m \cdot \frac{G_f \cdot (1 + f) + G_m \cdot (1 - f)}{G_m \cdot (1 + f) + G_f \cdot (1 - f)}$$

We do the same for the strength parallel, X(f), and perpendicular, Y(f), to the fibres and implement the Tsai-Hill maximum strain energy criteria of failure for multi-axial loading, recalling the the stress is actually a vector of thr stress, the stress parallel and perpendicular to the testing direction and the shear stress: we'll use the built-in **m** function to pick the appropriate minimum strain parallel to the fibres and minimum stress perpendicular to the fib

$$X(f) := E_x(f) \cdot \min(\varepsilon_m, \varepsilon_f)$$

$$Y(f) := \min(\sigma_m, \sigma_f)$$

$$\mathrm{Fail}(\sigma, f) := \frac{(\sigma_1)^2}{X(f)^2} - \frac{\sigma_1 \cdot \sigma_2}{X(f)^2} + \frac{(\sigma_2)^2}{Y(f)^2} + \frac{(\sigma_3)^2}{\left(\frac{Y(f)}{2}\right)^2}$$

We now set up the compliance matrix for the composite parallel to the fibres using the simple models set out ab such that the compliance is solely a function of the volume fraction of fibres, f. The stiffness tensor **Q** is just the of the compliance tensor **S**!

$$S(f) := \begin{pmatrix} \frac{1}{E_x(f)} & \frac{-\nu_{12}(f)}{E_x(f)} & 0 \\ \frac{-\nu_{12}(f)}{E_x(f)} & \frac{1}{E_y(f)} & 0 \\ 0 & 0 & \frac{1}{G(f)} \end{pmatrix} \qquad Q(f) := S(f)^{-1}$$

We can define a rotation matrix, T, where □ is the rotation in the x-y plane from the SPECIALLY ORTHOTROP lamina (where the principal stresses are aligned parallel and perpendicular to the fibre axes) to the GENERALL ORTHOTROPIC 'x-y' lamina (where the principal stress are not aligned with the fibre axes). The R matrix is req convert tensor strain to engineering strain. Finally, we define the compliance (**S**$_c$) and stiffness matrices (**Q**$_c$) of composite at any angle such that the compliance and stiffness matrices are functions of both the fibre volume fr and the angle between the applied stress and the fibre axis.

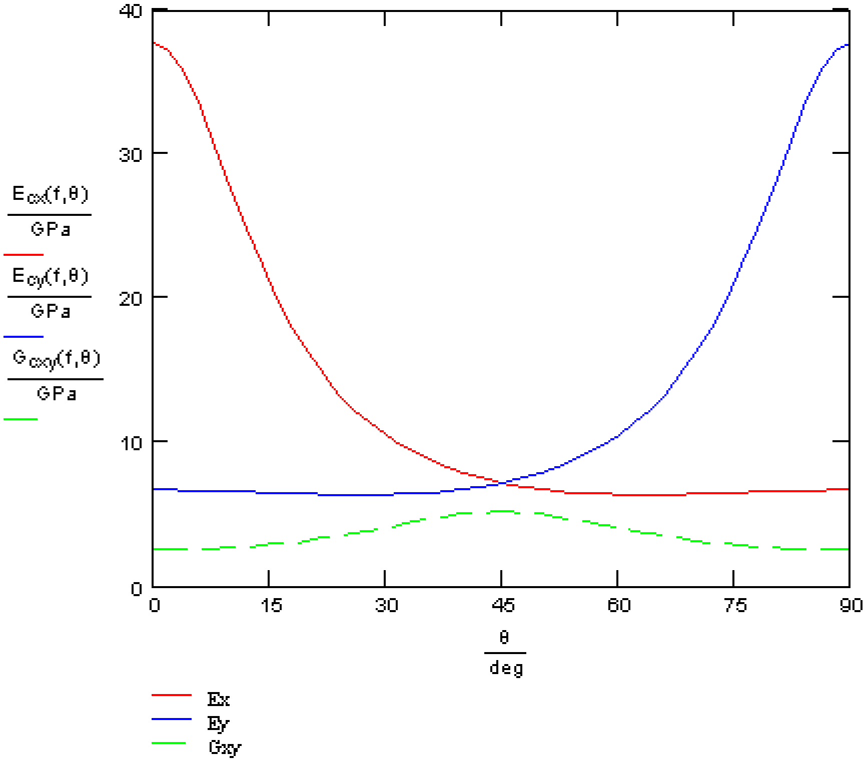
$$R := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad T(\theta) := \begin{pmatrix} \cos(\theta)^2 & \sin(\theta)^2 & 2 \cdot \cos(\theta) \cdot \sin(\theta) \\ \sin(\theta)^2 & \cos(\theta)^2 & -2 \cdot \cos(\theta) \cdot \sin(\theta) \\ -\cos(\theta) \cdot \sin(\theta) & \cos(\theta) \cdot \sin(\theta) & \cos(\theta)^2 - \sin(\theta)^2 \end{pmatrix}$$

$$Q_c(f, \theta) := T(\theta)^{-1} \cdot Q(f) \cdot R \cdot T(\theta) \cdot R^{-1} \qquad S_c(f, \theta) := Q_c(f, \theta)^{-1}$$

The 3 elastic modulii are then

$$E_{cx}(f, \theta) := \frac{1}{S_c(f, \theta)_{1,1}} \qquad E_{cy}(f, \theta) := \frac{1}{S_c(f, \theta)_{2,2}} \qquad G_{cxy}(f, \theta) := \frac{1}{S_c(f, \theta)_{3,3}}$$

which can be plotted out as

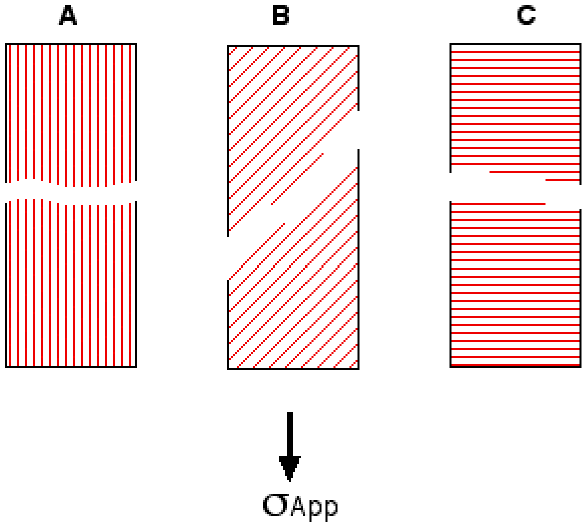


So we can now see that the elastic modulus of a composite in which all the fibres are aligned in a single directio off rapidly after about 3 or 4° of misalignment. Notice how MathCAD combines and hides all the matrix algebra v defining the 3 main elastic constants in terms of volume fraction and angle.

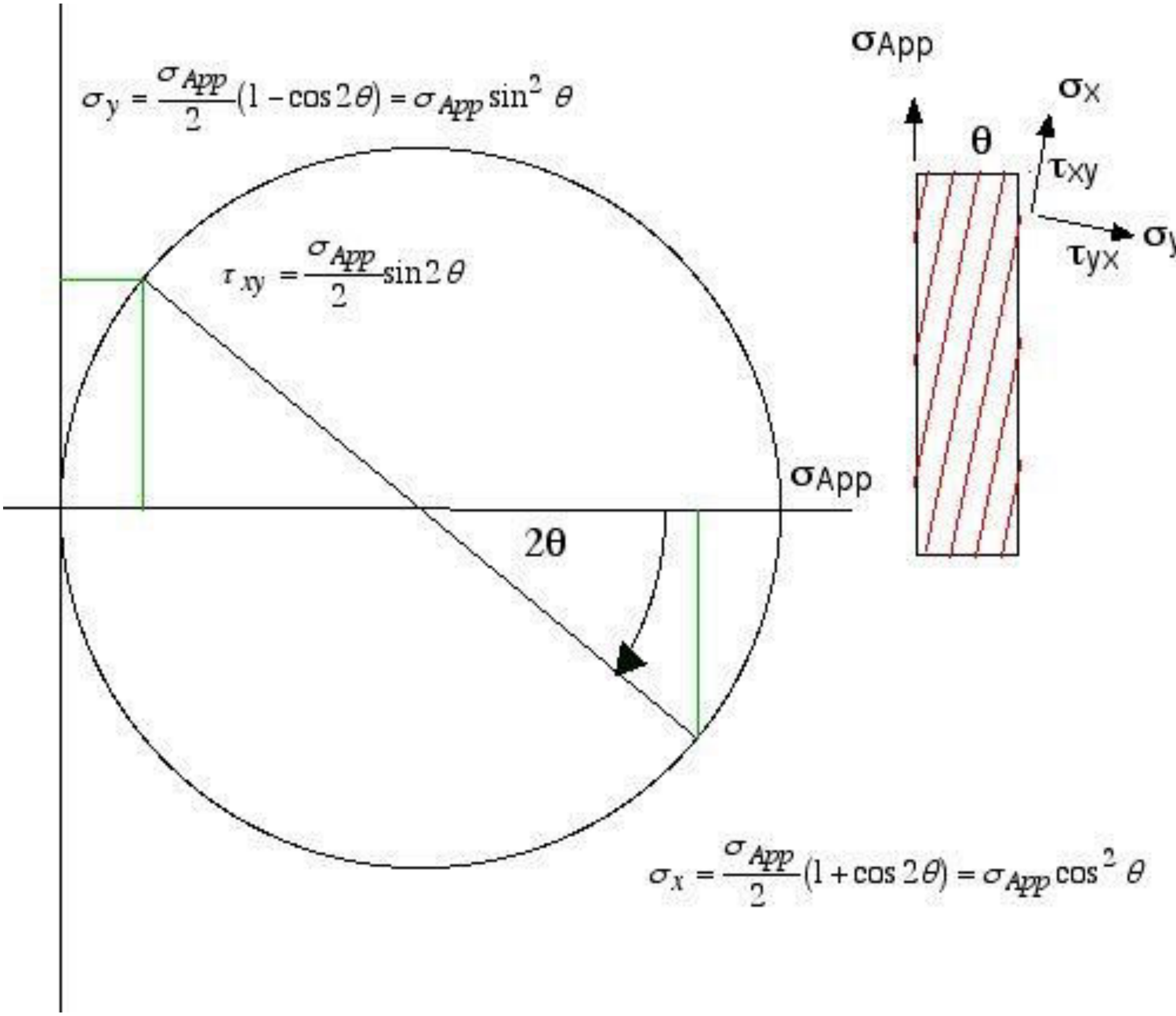
Effect of Fibre Orientation on the Strength of Aligned Continuous Fibre Composites

When considering the effect of fibre orientation on the strength of a composite material made up of a continuous fibres embedded in a matrix, it should be recognised that there are 3 possible modes of failure...

1. Tensile fracture parallel to the fibres (whether the fibres fail or the matrix fails will depend on the particula combination of fibre and matrix materials as well as the volume fraction of fibres),
2. Shear failure of the matrix as a result of a large shear stess acting parallel to the fibres ,
3. Tensile failure of the matrix or fibre/matrix interface when stressed perpendicular to the fibres.



We have already determined suitable expressions for the strength of a composite when tested parallel to the fib
We'll call this strength X. We also know the tensile strength of the matrix material which we'll call Y. The shear s
of the matrix can be determined using the Tresca criteria and is simply Y/2. In order to examine the effect of orie
on strength we need to make use of Mohr's Circle to establish the state of stress aligned parallel and perpendic
the fibres and then to equate these stresses with the appropriate failure stress of the composite in each those directions.

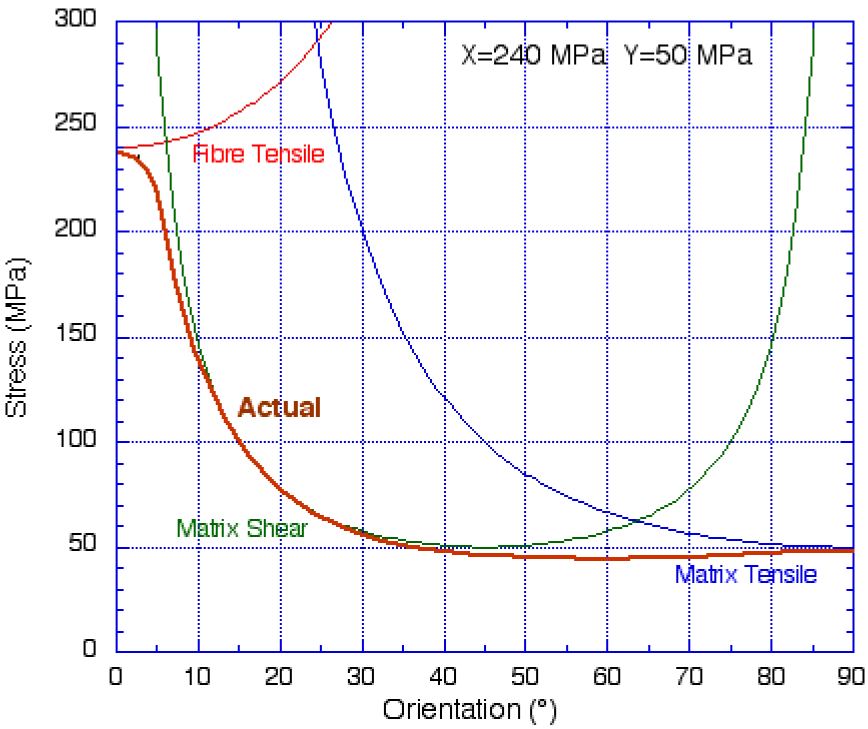


For failure to occur, the applied stress must be increase until either

$$\sigma_x > X \quad \sigma_y > Y \quad \tau_{xy} > \frac{Y}{2}$$

$$\sigma_{App} > \frac{X}{\cos^2\theta} \quad \sigma_{App} > \frac{Y}{\sin^2\theta} \quad \sigma_{App} > \frac{2Y}{\sin 2\theta}$$

These equations are plotted out below and since failure is a "weakest link" phenomenon, fracture will occure at whichever criterion is reached first and so the mechanism of failure changes from tensile failure of the fibres to s teh matrix to tensile failure of the matrix as the fibre angle is increased from 0 to 90°.



Failue under Mutliaxial Stress States (Plane Stress) - Tsai-Hill

When two mutually perpendicular stresses and/or a shear stress is applied to the composite we need to be able define a failure criterion. Tsai and Hill have established a suitable fracture criteria based on maximum strain ene rather than considering stress and strain. This maximum strain energy approach allows us to ignore the fact tha can occur because either a stress has exceeded a critical value (e.g. the stress resolved perpendicular to the fil exceeded the tensile strength of the matrix) or the strain has exceeded a particular value (e.g. the strain resolve parallel to the fibres has exceeded thefibre fracture strain). The Tsai-Hill maximum strain energy formulation is:-

$$\left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{2\tau_{xy}}{Y}\right)^2 - \left(\frac{\sigma_x \sigma_y}{X^2}\right) > 1$$

Which we can see for the case of a uniaxial stress applied

- Parallel to the fibres $\sigma_x = X \quad \sigma_y = 0 \quad \tau_{xy} = 0$,
- Perpendicular to the fibres $\sigma_x = 0 \quad \sigma_y = Y \quad \tau_{xy} = 0$,

- Simple shear $\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = Y/2$

is the result that we would expect.

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